Ecological Applications, 9(3), 1999, pp. 751–771


# MANAGEMENT OF EUTROPHICATION FOR LAKES SUBJECT TO POTENTIALLY IRREVERSIBLE CHANGE

S. R. Carpenter,[1,4] D. Ludwig,[2] and W. A. Brock[3]

[1]*Center for Limnology, 680 North Park Street, University of Wisconsin, Madison, Wisconsin 53706 USA*
[2]*Department of Mathematics, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z2*
[3]*Department of Economics, University of Wisconsin, Madison, Wisconsin 53706 USA*

*Abstract.* We analyzed management policies for ecosystems subject to alternate states, thresholds, and irreversible changes. We focused on the problem of lake eutrophication by excessive phosphorus (P) input. Eutrophic lakes may be classified, with respect to their response to reduced P input alone, as reversible (recovery is immediate and proportional to the reduction in P input), hysteretic (recovery requires extreme reductions in P input for a period of time), or irreversible (recovery cannot be accomplished by reducing P input alone). A model with one state variable and one control variable describes the responses of lake trophic state to changes in P input and other management interventions. Activities that generate P input to the lake are assumed to create profits, while the value of ecosystem services provided by the lake declines at high P levels. We then calculated P input policies that maximize the discounted net benefits from polluting activities and ecosystem services. If "optimality" is defined as maximizing this discounted criterion, then analyses based on deterministic lake dynamics usually lead to higher P input rates than analyses that assume various kinds of variability (e.g., inputs are affected by stochastic factors such as weather, policy is implemented with lags, or parameters of the limnological model are uncertain). In reality, all of these complications occur. Therefore, if maximum economic benefit is the goal of lake management, P input targets should be reduced below levels derived from traditional deterministic models. This pattern may apply to other situations where diffuse pollution causes nonlinear changes in ecosystem state, such as the greenhouse effect or acid deposition.

*Key words: economics, ecosystem management; ecosystems, alternate states; eutrophication; irreversible change; lake management; management policy, lakes; optimality; phosphorus input to lakes; policy analysis.*

## Introduction

Experience with management of large, complex environmental systems reveals patterns of ecosystem and human dynamics that seem to be repeated in case after case (Gunderson et al. 1995). Resilience, the capacity of a nonlinear system to remain within a stability domain (Holling 1973, Ludwig et al. 1997), is a central concept. Surprise and crisis are often the consequence when an ecosystem shifts between stability domains. Fishery collapse is a well-studied example (Walters 1986). Crisis may provide opportunities for learning, introduction of novel approaches, and reorganization, or it may prompt ever more rigid policies for ecosystem management (Gunderson et al. 1995). Paradoxically, rigid management systems can create conditions favorable to shifts in stability domains (Holling and Meffe 1996). Thus it seems important to understand the interactions of ecosystem resilience with social decision-making systems (Holling and Sanderson 1996).

The complex interactions of ecosystems and social systems evoke responses ranging from confusion and frustration, to calls for further research, to advocacy of particular management algorithms that purportedly solve the problems. Among these is an increasing interest in economic principles for ecosystem management (Gunderson et al. 1995, Perrings et al. 1995, Daily 1997).

The application of economics to ecosystem management raises a number of fundamental questions. How should alternate states, thresholds, and irreversible changes in the ecosystem affect economic policy prescriptions? These phenomena are central to ecosystem resilience, but they have not been addressed effectively in economic analyses. How should environmental stochasticity and uncertain ecological predictions affect economic analyses? Although there is an extensive literature on making decisions in the presence of uncertainty (e.g., Lindley 1985, Walters 1986), the implications for managing nonlinear ecosystems are not yet well understood.

This paper considers the consequences of alternate states and ecological variability for ecosystem management policies, according to economic analysis. To focus the issue, we address the particular case of lake eutrophication (Fig. 1). Two ecosystem states of lakes,

Manuscript received 3 March 1998; revised 18 September 1998; accepted 16 October 1998.
[4] E-mail: srcarpen@facstaff.wisc.edu

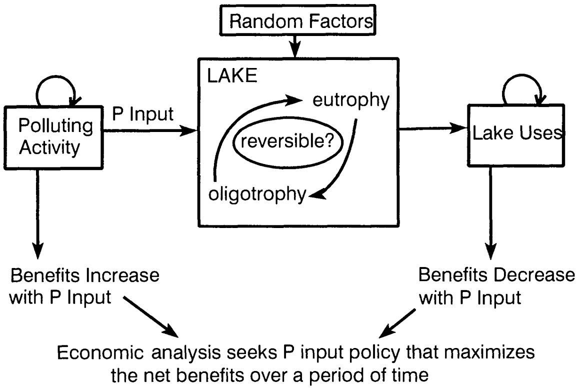


FIG. 1. Framework for the application of economic methods of policy choice to management of lake eutrophication.

oligotrophy and eutrophy, are of interest (Wetzel 1983, Carpenter and Cottingham 1997). Oligotrophic lakes are characterized by low nutrient inputs, low to moderate levels of plant production, relatively clear water, and relatively high value of ecosystem services. Eutrophic lakes have high nutrient inputs, high plant production, murky water, anoxia, toxicity, and relatively low value of ecosystem services. Thus eutrophication diminishes the net value derived from lakes (Postel and Carpenter 1997, Wilson and Carpenter 1999). The ultimate cause of eutrophication is excessive inputs of nutrients. Nutrient input is a by-product of activities such as agriculture, forestry, and urban development, and benefits from these activities are directly related to inputs. Thus there is a trade-off between benefits from polluting activities and costs of ecosystem services foregone due to consequences of pollution. We shall use economic analysis to compare policies for use and management that evaluate the net flow of benefits when there are trade-offs involved. How is the economic analysis affected by ecosystem dynamics, including difficulties in reversing eutrophication?

There are several advantages of focusing on this relatively simple example. (1) It is analogous to a number of other environmental issues in which benefits related to generation of pollutants must be balanced against loss of ecosystem services caused by pollution. Examples include global climate change, acid deposition, and other forms of water, air, and soil pollution. (2) We can use a model with one state variable and one control variable that is complex enough to represent alternate states, irreversibility, stochasticity, and uncertainty, yet is simple enough to be understood. (3) Data are available to estimate model parameters for real-world examples. (4) Because the model is relatively simple, it is relatively easy to catalog assumptions and omissions. These clarify needs and priorities for further work.

### *Overview of the paper*

We begin with a model for lake eutrophication. Our goal is a model that is consistent with the mechanisms of eutrophication and the state changes known from lakes, yet simple enough to be integrated with an economic model in a comprehensible way. We show that the model is supported by major patterns from comparative limnology, consistent with many case studies of lake eutrophication and its mitigation, and provides a reasonable fit to long-term ecological data.

Next, we develop an economic model and use it to calculate the policy that maximizes net economic value of polluting activities and ecosystem services under a variety of scenarios including stochastic inputs, lags in implementing policy, and uncertainty about lake response to changes in nutrient inputs. In general, stochasticity, lags, and uncertainty lead to policies that permit lower inputs than would be allowed by policies that ignore these factors. We summarize the implications of the model for limnologists and managers, the limitations of our approach, and some priorities for future work. Mathematical results are collected in the appendices.

## LAKE EUTROPHICATION

### *Description and scope of the problem*

Eutrophication, caused by excess inputs of nutrients, is a widespread and growing problem of lakes, rivers, estuaries, and coastal oceans (Smith 1998). In lakes, excessive inputs of phosphorus (P) are usually the primary cause (Schindler 1977). Negative effects of eutrophication include increased plant growth; shifts in phytoplankton to bloom-forming species that are often toxic or inedible; decreases in water transparency; problems with taste, odor and water treatment; oxygen depletion; and fish kills (Smith 1998). In the United States, eutrophication accounts for about half the impaired lake area, 60% of the impaired river reaches, and is the most widespread pollution problem of estuaries (NRC 1993, USEPA 1996). Most of the excess P input to U.S. waters is caused by nonpoint pollution (Carpenter et al. 1998*a*). The source of this pollution is runoff from agriculture and urban lands. Because sources are diffuse, this pollution is difficult to measure and regulate (Novotny and Olem 1994). Economic data indicate that losses due to nonpoint pollution, or benefits from nonpoint pollution control, are large (Postel and Carpenter 1997, Wilson and Carpenter 1999). For example, the benefits of federal policies intended to achieve "swimmable" water quality for all U.S. freshwaters are about U.S. \$5.8 $\times$ $10^{10}$ per year (Carson and Mitchell 1993, converted to 1997 dollars by Wilson and Carpenter 1999).

While nutrient addition may lead to immediate increases in symptoms of eutrophication, decreased nutrient input does not always cause immediate or complete reversal of eutrophication (Sas 1989, NRC 1992, Cooke et al. 1993). Explanations for delayed response, or lack of response, by lakes to reduced nutrient input focus on recycling of P. As lakes are enriched, P accumulates in sediments and rates of recycling from sediments to the overlying water ("internal loading") increase. Whole-lake experiments show that recycling rates can build to significant levels in a matter of of years (Schindler et al. 1987, Houser 1998). Culturally eutrophic lakes may receive excessive P inputs for decades or longer. On an annual basis, recycling from sediments to water of eutrophic lakes commonly exceeds inputs of P (Nürnberg 1984, Soranno et al. 1997). Recycling of P from sediments interferes with mitigation of eutrophication, and can sustain eutrophy long after external inputs of P are decreased (Sas 1989, NRC 1992, Cooke et al. 1993). In some cases, eutrophication cannot be reversed by decreasing P input alone (Larsen et al. 1979, 1981, NRC 1992, Cooke et al. 1993, Scheffer et al. 1993). In such lakes, additional interventions that decrease recycling, accelerate sedimentation, or increase outputs of P are needed (NRC 1992, Cooke et al. 1993). Success of these interventions often depends on site-specific characteristics such as lake area or mean depth, food web structure, or submerged vegetation. These interventions will not be feasible or effective in all lakes.

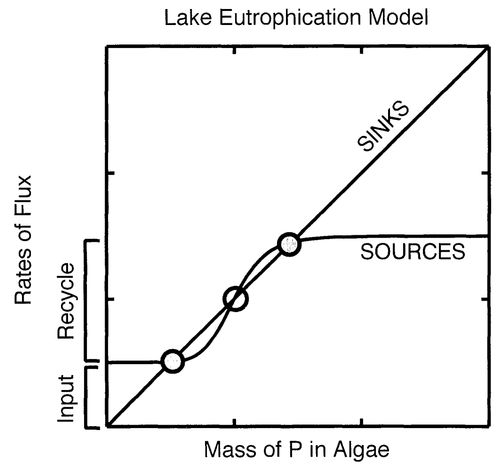


FIG. 2. Rates of P flux vs. P mass in the water, according to Eq. 1. The diagonal line is the rate of P loss. The sigmoid line represents the P sources (inputs + recycling). Intersections of these lines are the steady states. The open circle denotes the unstable steady state. Shaded circles denote stable steady states.

*Lake response to P input and recycling: a model*

The essential dynamics of lake eutrophication can be modeled by the equation

$$\frac{dP}{dt} = l - sP + \frac{rP^q}{m^q + P^q}. \qquad (1)$$

The dynamic variable, $P$, is the amount of P (mass or concentration) in the water column. The rate of P input (mass or concentration per unit time) from the watershed is $l$. The rate of P loss per unit time ($\mathrm{time}^{-1}$) is $s$. Loss processes include sedimentation, outflow, and sequestration in biomass of consumers or benthic plants. The maximum rate of recycling of P (mass or concentration per unit time) is $r$. P can be recycled from sediments or by consumers. In the subsequent analyses, we will assume that sediments are the major source of recycled P. The overall recycling rate is assumed to be a sigmoid function of $P$ represented by the last term in Eq. 1. The exponent $q$ (dimensionless; $q \geq 2$) affects the steepness of the sigmoid curve at the point of inflection. Larger values of $q$ give a steeper curve. The $P$ value (mass or concentration) at which recycling reaches half the maximum rate is $m$.

The lake has alternative states of oligotrophy or eutrophy for certain values of the parameters (Fig. 2). These can be visualized by overlaying plots of the straight line for P loss processes (sinks) and the curve for P sources (input + recycling). For certain parameter values, the curves intersect at three points. The central point is an unstable repeller that lies between two stability domains (Appendix A). The upper and lower intersections are attractors, one for eutrophy and the other for oligotrophy. Some extrinsic manipulation is necessary to flip the lake from one domain of attraction to the other.

For this model to be useful, major P fluxes must depend on the amount of P in the water column roughly as depicted in Fig. 2. Specifically, (1) P sinks must be directly proportional to the amount of P in the water, (2) P recycling must increase in an approximately sigmoid manner (low slope when P levels are low or high, with highest slope at an intermediate P level), and (3) in at least some circumstances the slope of the recycling curve must exceed the slope of the line for sink processes. We address these three conditions in turn.

*Sinks proportional to P.*—P is lost from the phytoplankton by outflow, sedimentation, buildup of dissolved or detrital P in the water column, or accumulation in other organisms such as fishes. Although many factors can affect the rates of these loss processes, all of them are proportional to the mass of material in the water column. The largest losses are typically sedimentation and outflow (Frisk et al. 1980, Nürnberg 1984, Lathrop et al. 1998). Both of these rates are directly related to mass (or concentration) in the water column (Reckhow and Chapra 1983, Baines and Pace 1994). This is the basis for the linear approximation $-sP$, which represents losses in Eq. 1.

*Sigmoid P recycling.*—Case studies of eutrophication management (Sas 1989, NRC 1992) and cross-lake comparisons (Nürnberg 1984) show that recycling of P from sediments is proportional to the amount of P in the water and depends on the history of P inputs. Some fraction of the recycling is due to resuspension of sediments by waves or benthivorous fishes. Sediment resuspension may be the major mechanism of recycling in shallow unstratified lakes. In stratified lakes, recycling also depends on oxygen depletion in

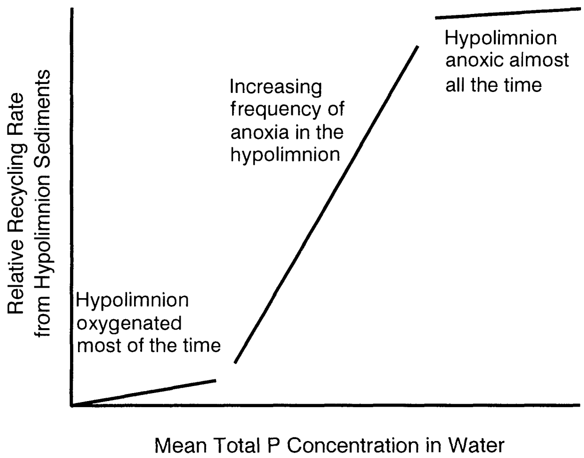


FIG. 3. The general pattern expected for P recycling from the hypolimnion vs. total P mass or concentration in the water.

the hypolimnion during the stratified season (Mortimer 1941). Interlake variation in this mechanism is correlated with sulfate concentration, which apparently affects the availability of iron to bind P (Caraco et al. 1991). When the hypolimnion is oxygenated, iron exists in its oxidized state. It forms insoluble complexes with P that prevent recycling. When the hypolimnion is deoxygenated, iron is reduced and inorganic P becomes soluble, thereby increasing the recycling rate. Oxygen depletion in the hypolimnion is driven by decomposition of sinking phytoplankton and respiration of surface sediments. Both the flux of sinking particles and water column respiration are proportional to P concentration and primary production (Baines and Pace 1994, del Georgio and Peters 1994). Primary production increases with the amount of P in the water (Schindler 1978).

The probability of anoxia in lakes is a sigmoid function of $P$ (Reckhow 1979). This probability is correlated with the number of days that sediment is overlain by anoxic water (Nürnberg 1995).

According to these findings, recycling should increase with P following the general pattern of Fig. 3. When P is low, the hypolimnion is oxygenated most of the time. Recycling from the hypolimnion is limited to mineralization of P from sinking phytoplankton that were actively growing just days or weeks before. Also, there is some resuspension of sediments, especially in shallower lakes, but the recycling rate is not great because the sediment P concentration is low. When P is high, the hypolimnion is anoxic almost all the time and recycling from the hypolimnion is near maximal. At intermediate P levels, recycling is increasing the most rapidly with P. The frequency of anoxia in the hypolimnion is growing, so on an annual basis the hypolimnion is recycling P for an increasing proportion of the available time.

*P recycling can be faster than retention.*—Annual P recycling from the hypolimnion of a stratified lake is roughly proportional to the amount of time that the hypolimnion is anoxic. How does recycling rate change with $P$? Comparative limnology suggests answers to this question.

The rate of oxygen depletion in the hypolimnion depends on P concentration of the lake water, mean thickness or depth of the hypolimnion, and temperature of the hypolimnion. Cornett and Rigler (1980) presented a regression that predicts oxygen depletion rate from P concentration and mean lake depth. Charlton (1980) presented a regression that predicts oxygen depletion rate from chlorophyll concentration, hypolimnion thickness, and hypolimnion temperature. Chlorophyll, a measure of phytoplankton biomass, is proportional to P concentration (Canfield and Bachmann 1981). The Cornett–Rigler and Charlton models employ somewhat different approaches and both will be considered here.

The proportion of anoxic days during the stratified season increases with P concentration under both models (Fig. 4). Anoxia is more prevalent as lakes become shallower and the hypolimnions become warmer. These calculations assume that the stratified season is 150 d long and that the initial hypolimnetic oxygen concentration is 12.7 mg/L, the saturated concentration at 4°C.

The curves of Fig. 4 are proportional to recycling rates from the hypolimnion. We assumed that recycling was zero when the hypolimnion was oxic. When the hypolimnion was anoxic, we assumed that recycling occurred at the median rate reported by Nürnberg (1984), 12 $mg \cdot m^{-2} \cdot d^{-1}$. With these assumptions, the curves of Fig. 4 can be converted to curves of annual P recycling vs. mean total P concentration in the water. Slopes of these curves have units $yr^{-1}$, as do P retention rates (Frisk et al. 1980, Nürnberg 1984). We averaged slopes over a P interval of 20 $\mu g/L$ starting from the P level that produced just one anoxic day per year. This procedure underestimates the maximum slope, because slopes are averaged over a broad interval of P concentrations. Thus the slopes can be viewed as minimal estimates of the true slope (Fig. 5).

Can the minimal estimates of the recycling slopes (Fig. 5) exceed $s$? A precise estimate of $s$ would require measurements of P sedimentation, outflow, consumption, and recycling at short time intervals through the annual cycle. However, an estimate of $s$ can be calculated from the annual carryover of P in the lake water which is $\sim e^{-s}$ (Appendix A). In experimental lakes that are fertilized during summer for a period of years, P concentrations typically return to a baseline level each spring (Schindler et al. 1978, 1987, Carpenter et al. 1998*b*, Houser 1998). P levels rise with summer fertilization, then decline during fall and winter, approaching the baseline level in spring. The spring P concentration is ~5–20% of the maximum concentration attained the preceding summer (Houser 1998; S. R. Carpenter, *unpublished data*). If we take this percentage as a measure of carryover, then $s \approx -\log(0.05)$ to $-\log(0.2) \approx 1.6$ to 3. We expect $s$ to be greatest in

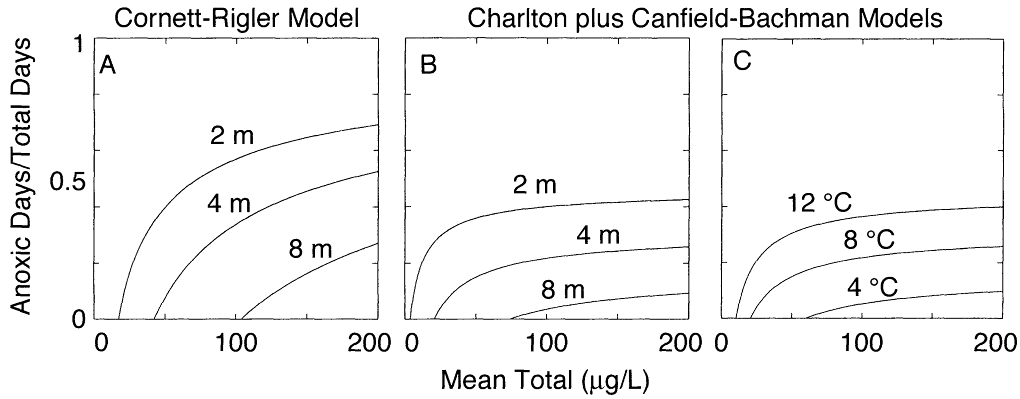


FIG. 4. Predictions of the proportion of anoxic days vs. mean total P concentration (μg/L) from published empirical relationships. All calculations assume a 150-d stratified season. (A) Model of Cornett and Rigler (1980) for lakes of three mean depths. (B) Model of Charlton (1980) combined with the P–Chl relationship of Canfield and Bachmann (1981) for lakes of three mean hypolimnion thicknesses at 8°C. (C) Charlton model for lakes at three hypolimnetic temperatures with a mean hypolimnetic thickness of 4 m.

lakes with rapid flushing and rapid sedimentation (low carryover of P), and lowest in lakes with slow flushing and sedimentation (high carryover of P).

These calculations suggest that the slope of the recycling curve will substantially exceed $s$ under some conditions, leading to alternate states. Recycling slopes are greater for shallower lakes, suggesting that the occurrence of alternate states by this mechanism should be more common in shallower lakes. Our findings corroborate Scheffer's (1998) view of alternate states in shallow lakes. Recycling slopes are also greater for lakes with warmer hypolimnions, suggesting that alternate states should occur more commonly as lakes become warmer. Lakes with slow flushing and effective mixing by the wind (which suspends P, leading to slow sedimentation rates) should have low values of $s$ and therefore exhibit alternate states.

The empirical limnological models also provide a rough estimate of $q$, the exponent of the recycling curve. The slopes of Fig. 5 approximate the derivative of the sigmoid curve evaluated at $m \approx P$, which is $qr/4P$. Therefore $q \approx$ slope $\times$ $4P/r$ where $P$ is the P concentration near the steepest part of the recycling curve. Values of $q$ range from ~20 for shallow, warm lakes to ~2 for deep, cold lakes.

### *Application of the model to eutrophication and its mitigation*

So far we have provided evidence for the general patterns and limnological mechanisms described by the model. Another way to assess the model's usefulness is to ask whether it can describe common experiences with lake eutrophication and mitigation. Here we explain how several common scenarios are represented in the model.

Eutrophication is caused by increasing P inputs (Schindler 1977). In the model, this process is represented as the transition from the lower to the upper sigmoid curve in Fig. 6. With the lowest P input, there is one steady state at relatively low P. With the intermediate P input, there are three steady states, two of them stable. One of the stable steady states is at low

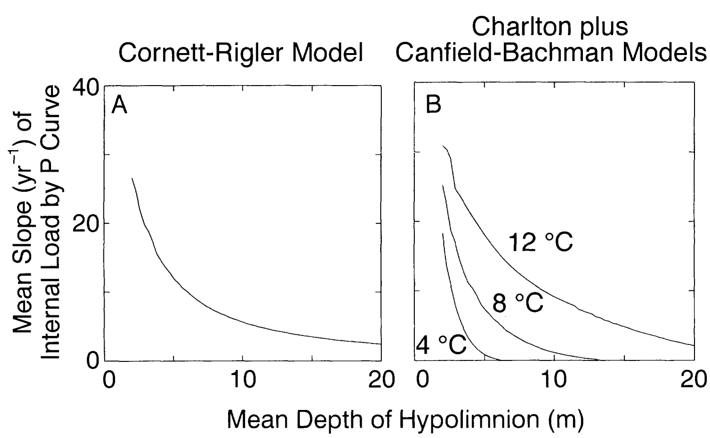


FIG. 5. Mean slopes (yr$^{-1}$) of the recycling curve (internal load vs. P curve) near the steepest point. (A) Slope vs. mean depth from the Cornett and Rigler (1980) model. (B) Slope vs. hypolimnion mean thickness at three hypolimnion temperatures from the Charlton (1980) model using the P–Chl relationship from Canfield and Bachmann (1981).

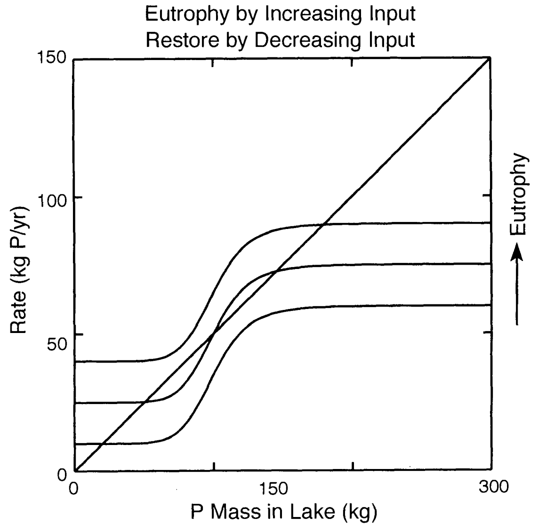


FIG. 6. Lake eutrophication or restoration by manipulating P inputs, according to Eq. 1. The diagonal line is the P sink and sigmoid lines are the P sources. Increasing P input moves the system from the low-P steady state to the intermediate sigmoid curve with two stable steady states, and then to the upper curve with only the high-P steady state. Restoration reverses the sequence.

P, the other at high P. With the highest P input, there is again only one steady state, this one at relatively high P.

Reduction of P inputs is generally believed to be essential for mitigation of eutrophication (NRC 1992, Cooke et al. 1993). In the model, restoration by reduced P input is represented by transition from the upper to the lower sigmoid curve in Fig. 6.

In fact, decreases in P input may not have an immediate effect on P levels in the lake (Sas 1989, NRC 1992, Cooke et al. 1993). Additional interventions such as sediment treatment, hypolimnetic oxygenation, or biomanipulation may be needed to restore the lake (NRC 1992, Cooke et al. 1993). This situation is represented by the intermediate sigmoid curve with two steady states (Fig. 6). As the system moves from high P inputs to intermediate P inputs, the P level will remain near the high-P steady state. The interventions push the P level below the unstable steady state to the region where P can approach the lower steady state.

Reductions in P input may have a delayed effect, so that decreased P in the lake does not occur until some years after input is reduced (Sas 1989). In terms of the model, recycling rates increase during an extended period of high P input (Fig. 7). When inputs are reduced, recycling remains high and sustains the eutrophic state. Over a period of years, nutrients in surface sediments may become depleted, recycling declines, and the lake returns to the low-P steady state.

Some lake restoration methods involve interventions to reduce recycling. For example, air or oxygen may be injected into the hypolimnion to reduce recycling and shift a lake from a high-P steady state to a low-P steady state (NRC 1992, Cooke et al. 1993, Prepas and Burke 1997). In shallow lakes, macrophyte restoration stabilizes sediments and reduces recycling (Scheffer et al. 1993). In terms of the model, such interventions are represented as a compression of the recycling curve (Fig. 8A).

Other lake restoration methods involve interventions to increase sinks for P. Biomanipulation transfers nutrients from phytoplankton to consumers and sediments (Carpenter et al. 1992, Schindler et al. 1995). Aluminum sulfate can be added to lakes to precipitate P to the sediments (Cooke et al. 1993). In reservoirs, flushing rate can be manipulated to increase losses of nutrients (Cooke et al. 1993). In terms of the model, these restoration methods are represented as a steepening of the P sink line (Fig. 8B).

## ALTERNATE STATES AND IRREVERSIBILITY

Although reductions of P input are believed to be necessary for mitigation of eutrophication, experience shows considerable variability in lake responses to reduced P inputs (Ryding 1981, Cullen and Forsberg 1988, Jeppesen et al. 1991, NRC 1992, Cooke et al. 1993). On the basis of these experiences, we classify lakes as "reversible," "hysteretic," or "irreversible" with respect to their response to reductions in P input, and show how these types are represented by the model.

### *Reversible lakes*

In some lakes, eutrophication can be reversed by P input controls alone. Lake Washington (Washington, USA) (Edmondson 1991) is a familiar example. Lakes that are deep and cold, with rapid flushing or sediment chemistry favorable to P retention, or lakes that have been eutrophied for only a short time may be restored

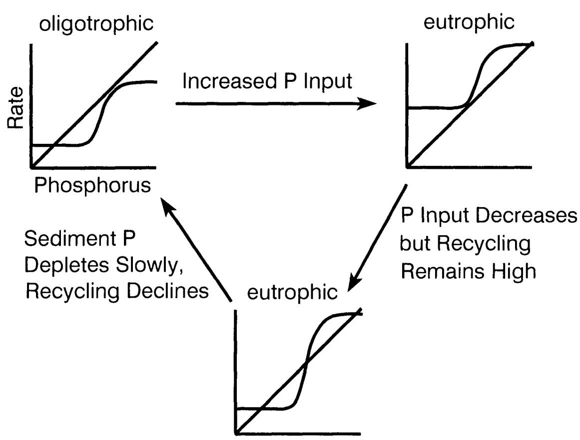


FIG. 7. Explanation of delayed response to P input reduction using the model. Increased P input converts an oligotrophic lake to a eutrophic lake. Management decreases P input, but recycling remains high. Eventually sediment P is depleted, recycling declines, and again the lake becomes oligotrophic.

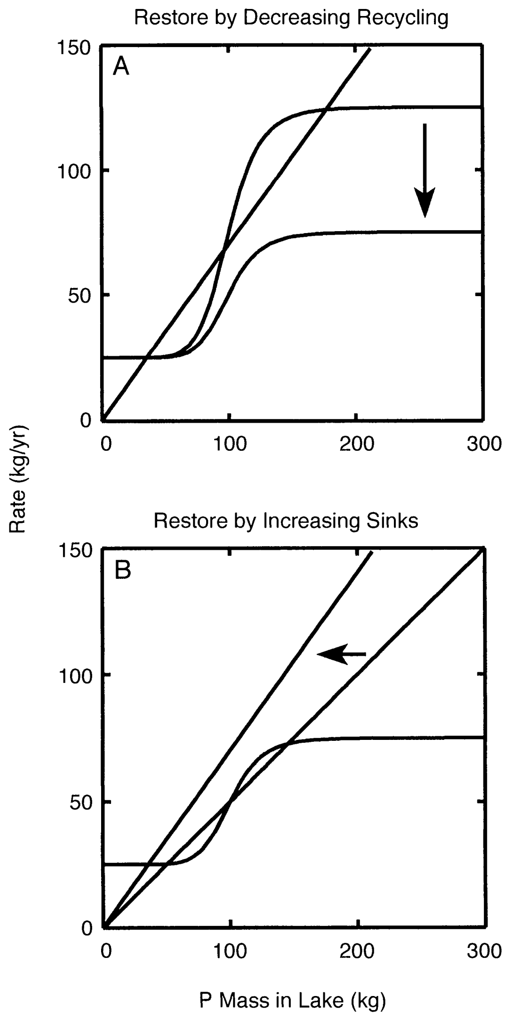


FIG. 8. (A) Lake restoration by decreasing recycling. (B) Lake restoration by increasing sinks.

rapidly by reducing P input. As represented by the model, eutrophication can be reversed by P input controls alone if the slope of the recycling curve is less than the slope of the line for P losses (Fig. 9A). Such lakes will be called "reversible."

### *Hysteretic lakes*

In some lakes, eutrophication has been reversed by combining P input controls with temporary interventions such as chemical treatments to immobilize P, or biomanipulation. These lakes are often small and/or shallow. Shallow lakes are more likely to have rapid P recycling, and therefore be unresponsive to P input controls alone. In smaller lakes, interventions such as aluminum sulfate treatment and biomanipulation are more feasible. In terms of the model, such lakes are shifted from a high-P steady state to a low-P steady state (Fig. 9B). Lakes that can be restored by perturbing them to the lower P steady state will be called "hysteretic." The qualitative behavior of hysteretic lakes in our model is similar to the more detailed theory for shallow lakes developed by Scheffer (1998).

### *Irreversible lakes*

In some lakes, eutrophication has not been reversed even by severe reductions in P input. The minimum P input to a lake is determined by regional factors, such as soil chemistry and airborne P deposition, that are not susceptible to management (NRC 1992). The minimum attainable P input may not be low enough to shift the lake out of the eutrophic state. This situation is most likely to occur in shallow lakes, lakes in P-rich regions, or lakes that have received extreme P inputs for an extended period of time (Jeppesen et al. 1991, NRC 1992). Shagawa Lake, Minnesota, USA, is a well-documented example of a stratified lake with apparently irreversible dynamics (Larsen et al. 1979, 1981). In terms of the model, recycling is so large that no feasible reduction of inputs can move the lake to the low-P steady state (Fig. 9C).

Typically, the minimum possible P input rate will be greater than zero, not zero as depicted in Fig. 9C. Lake managers recognize that the best attainable water quality for a lake is related to the minimum attainable P input rate dictated by watershed geology, soils, and

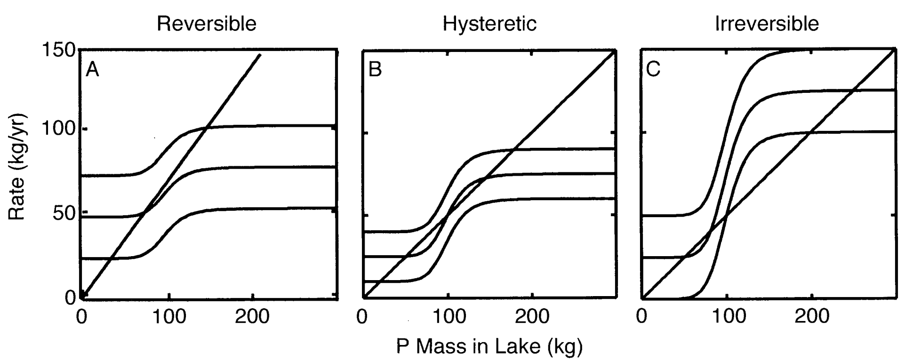


FIG. 9. P sources (sigmoid curves) and sinks (diagonal lines) for (A) reversible lakes, (B) hysteretic lakes, and (C) irreversible lakes.

hydrology (NRC 1992, Cooke et al. 1993). As the minimum attainable P input increases, it becomes more likely that eutrophication cannot be reversed by controlling P input alone.

In this paper, irreversible lakes are those that cannot be restored by P input controls alone. In principle, such lakes may be restorable by other interventions. However, there is no guarantee that other interventions will be possible. For example, biomanipulation is difficult in heavily fished lakes (Kitchell 1992) and in lakes with large surface areas, oxygenation and aluminum sulfate treatment are extremely costly (Cooke et al. 1993).

## Economic Framework for Managing Lake Eutrophication

Economic analysis seeks to maximize the net value to society of profits from activities that add P to the lake (such as agriculture or development) and the ecosystem services provided by the lake (such as clean water, recreation, and so forth). The notion of cost-benefit accounting for nonmarket environmental goods and services is detailed in a number of texts (e.g., Freeman 1993, Hanley and Spash 1993, Dixon et al. 1994). Goulder and Kennedy (1997) provide a brief, well-written summary.

In the most common economic approach to decision making, the ''optimal'' policy is defined as the one which maximizes the discounted net present value of a time series of benefits. This notion of optimality is debated (Bromley 1990). We wish to explore the consequences of this common definition of optimality for management of a lake subject to nonlinear dynamics and other complications including stochastic P inputs, lags in implementation of policy, and uncertain estimates of limnological parameters.

### *Model for economic utilities*

In the economic analyses, we use a discrete form of the lake model (Appendix A). The dimensionless dynamic variable is $X = P/m$. The discrete model is

$$X_{t+1} = X_t + a - bX_t + f(X_t) \tag{2}$$

where $a$ and $b$ are functions of the original parameters (Appendix A) and

$$f(X) = \frac{X^q}{1 + X^q}. \tag{3}$$

In this form of the model, $X$ is the state variable to be managed, $a$ is the input which is the single control variable, and $b$ is a parameter that determines whether the lake is reversible, hysteretic, or irreversible for a given value of $q$.

The utility of activities that cause P input ($U_L$) to the lake is calculated using information on the amount of money emitters are willing to pay for the P they release (Fig. 10A). Each emitter is ranked by willingness to pay, and the area under the curve from zero to $a$ is the utility of P input at rate $a$. If the willingness

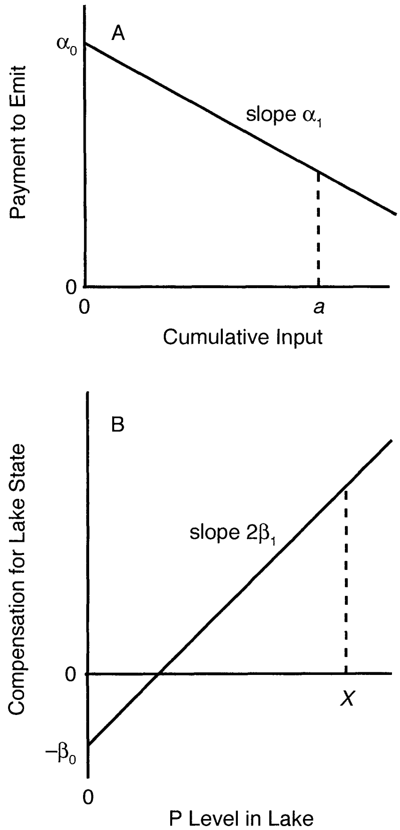


FIG. 10. (A) Willingness to pay for an increment of P emission (dollars per unit input) vs. total P input to the lake (dimensionless input $a$). (B) Compensation required to accept a level of P in the lake (dollars per unit P in the lake) vs. P level in the lake (dimensionless, $X$).

to pay curve is linear with intercept $\alpha_0$ and slope $\alpha_1$, then

$$U_L(a) = \alpha_0 a - \frac{\alpha_1 a^2}{2}. \tag{4}$$

Eq. 4 represents the demand for the right to pollute. The ''supply'' of tolerable pollution is calculated from information on the amount of compensation lake users are willing to accept to tolerate a given state of the lake, or the amount of money lake users would be willing to pay to remove an increment of P from the lake (Fig. 10B). The ordinate of Fig. 10B represents a loss to society as a whole. This sum of money will rise with P level, and cross the $x$-axis at some positive $X$ that represents the amount of P needed to support ecosystem services such as fish production. Below this level of $X$, lake users would be willing to pay emitters for more P; above this level, lake users would want to be compensated for tolerating excess P. The total (negative) cost of a given level of phosphorus $X$ is the negative of the integral of the curve in Fig. 10B from zero to $X$. If we assume that cost grows linearly with $X$, then the total (negative) cost $U_P$ is

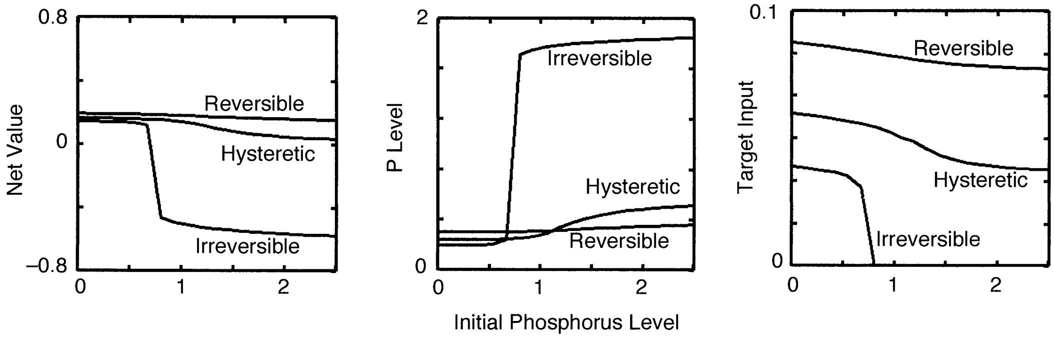


FIG. 11. Utility (net value), P level (dimensionless), and target P input rate (dimensionless) for a reversible lake ($b$ = 0.62), a hysteretic lake ($b$ = 0.52), and an irreversible lake ($b$ = 0.42) vs. initial dimensionless P level for lakes managed to maximize discounted net present value. In these simulations, $q = 2$, $\alpha_0 = 0.4$, $\beta_1 = 0.08$, $\delta = 0.98$.

$$U_P(X) = \beta_0 X - \beta_1 X^2. \quad (5)$$

The net total value of an input $a$ that leads to lake state $X$ is $U_L(a) + U_P(X)$.

In general, the total value will be high when inputs are low (so that $U_L(a)$ is positive) and P levels are low (so that $U_P(X)$ is positive). Total value will be low when inputs are high (so that $U_L(a)$ is small) and P levels are high (so that $U_P(X)$ is a large negative value). The $U_L(a)$ and $U_P(X)$ curves cross at some intermediate level of $a$ and $X$. This general pattern leads to the results described in the remainder of the paper. To produce this pattern using a minimum number of parameters, we assume $\alpha_1 = 0$ and $\beta_0 = 0$. This assumption has no effect on our conclusions about effects of stochastic inputs, lags in implementing policy, uncertainty, or discounting. In an analysis for a specific lake, one would construct curves like Fig. 10 and estimate site-specific parameters for $U_L$ and $U_P$ (see Appendix B).

### *Policy choice to maximize utility*

We will consider the policy problem of finding the P input level $u(X_{t-\tau})$ that will maximize a time series of benefits (Eqs. 4 and 5) subject to ecosystem dynamics given by

$$X_{t+1} = X_t + u(X_{t-\tau}) - bX_t + f(X_t) \quad (6)$$

where the parameter $b$ sets the P loss rate and $f(X_t)$ is given by Eq. 3. The input rate targeted by the policy depends on the level of P, $\tau$ time steps in the past. Except where otherwise stated, the lag $\tau$ is zero in the calculations reported below.

Given a policy for setting P input levels, the discounted present utility of a sequence of states from the present ($t = 0$) to some future time ($t = F$) is

$$V(u) = \sum_{t=0}^{t=F} \delta^t [U_P(X_t) + U_L(a_t)]. \quad (7)$$

The discount factor, $\delta$, is used to adjust future utilities to present utility. The "optimal" policy is defined to be the one that maximizes $V(u)$ (Appendix C).

## DETERMINISTIC MODEL, NO LAGS

Both the initial state of the lake and lake dynamics have significant effects on policy choice (Fig. 11). The curves show average utility, P levels, and P input rates for the policy that maximizes discounted present utility (Eq. 7) for specified values of the parameters. For a reversible lake, policy choice is not sensitive to initial P. Target load decreases slightly as initial P increases, but mean P level and net utility are nearly constant. For the irreversible lake, however, the optimal policy is very sensitive to the initial state of the lake. If the intial P concentration is low, the lake can be maintained in the oligotrophic state by moderating P inputs. If the initial P concentration is higher, however, the lake cannot be shifted to the oligotrophic state by reducing P inputs alone. Utility is positive at low P concentrations, negative at high P concentrations. The hysteretic lake is an intermediate case: it is more sensitive to initial conditions than the reversible lake, but less sensitive than the irreversible lake.

Implications for lake management policy are familiar. The utility of lakes that are initially oligotrophic can be maintained by policies that limit P input. The model indicates that reductions in P input are the optimal policy for lakes that are initially eutrophic. For eutrophic lakes with irreversible dynamics, no P input policy can restore the lake and yield positive utilities. Lake managers faced with this situation would consider additional interventions, beyond reductions in P input, to restore the lake (Cooke et al. 1993). Economic losses due to poor water quality would be part of the rationale for these interventions.

## STOCHASTIC INPUTS

Industrial or municipal discharges of nutrients to lakes may vary little from year to year. Nonpoint inputs (such as agricultural or urban runoff), however, are often the major cause of eutrophication and are highly variable in time (NRC 1992, Novotny and Olem 1994, Carpenter et al. 1998*a*). How should stochasticity of P inputs affect policy choice?

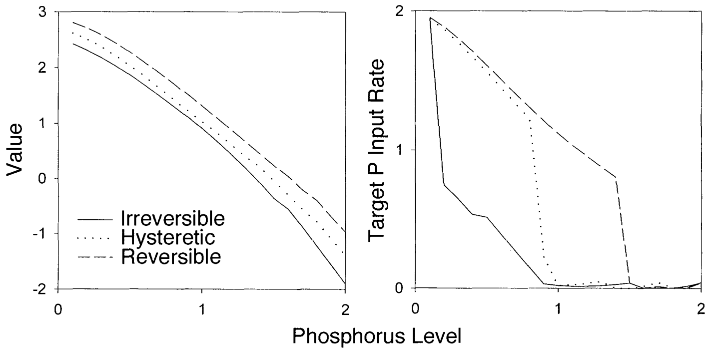


FIG. 12. Utility (value) and target P input rate (dimensionless) vs. P level (dimensionless) for reversible ($b$ = 0.62), hysteretic ($b$ = 0.52), and irreversible ($b$ = 0.42) lakes with stochastic inputs, managed to maximize discounted net present value. Values of the other parameters: $q$ = 2, $\alpha_0$ = 1, $\beta_1$ = 0.5, $\delta$ = 0.98, $\sigma$ = 0.25, df = 10.

*Stochastic form of the model*

We included stochasticity in the discrete model as $e^z$ where $z$ is a random variable from a Student's $t$ distribution with specified scale factor and degrees of freedom:

$$X_{t+1} = X_t + u(X_t)e^z - bX_t + f(X_t). \quad (8)$$

According to Eq. 8, P inputs are log-$t$ distributed (or lognormal if degrees of freedom are large) which is commonly the case in nature (Reckhow 1979). Computation of the optimal policy for the stochastic model is described in Appendix D.

*Policy choice with stochastic input*

As initial P increases, the target input rate decreases and the net utility decreases (Fig. 12). The decrease is steepest for the irreversible lake indicating a more cautious policy. Because inputs are stochastic, there is a chance that a high input will flip the lake to the eutrophic state. This risk is built into the utility calculation, and leads to lower P inputs if the lake is irreversible.

How does optimal policy change as inputs become more variable? Scenarios of high variability and low variability are compared for an irreversible lake in Fig. 13. These scenarios differ in both the scale factor of the $t$ distribution, which increases the spread of the distribution, and the degrees of freedom, which are inversely related to the thickness of the distribution's tails. Magnitude of variability has little effect on the utilities because the center of the distribution is unchanged. However, there is a marked effect on target input rates. When variability is low, input rates decline smoothly to near zero as the lake approaches the threshold for flipping to the high-P state. When variability is high, the decline in input rate is steeper and occurs at a relatively low P value because of the risk that an extreme input event could flip the lake to the high-P state.

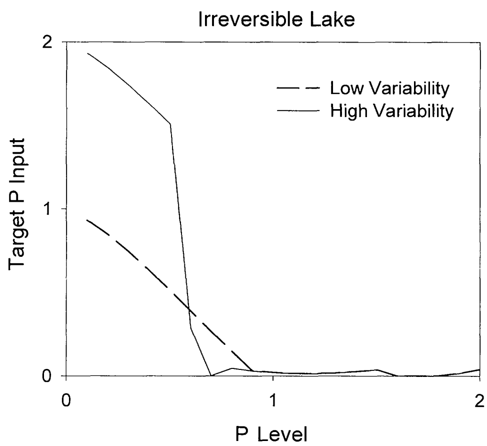


FIG. 13. Target P input rate (dimensionless) vs. P level (dimensionless) for scenarios in which inputs have low variance ($\sigma$ = 0.1, df = 25) and high variance ($\sigma$ = 0.3, df = 5). In both scenarios the lake is irreversible ($b$ = 0.42, $q$ = 2) and managed to maximize discounted net present value.

## EFFECTS OF LAGS

Lags in implementing P input policy tend to destabilize the ecosystem (Fig. 14). If there is no lag, input rates and P concentrations reach target levels in one time step (Fig. 14A). If there is a 10-yr lag in implementing policy, but this lag is ignored in setting policy, both P levels and input rates fluctuate considerably over time (Fig. 14B).

One way to compensate for the effect of lags is to implement policy gradually, so that the target P level is reached over a number of years (Appendix C). Gradual control greatly reduces the fluctuations and in the long run yields P concentrations and loading rates comparable to the situation with lag = 0 (Fig. 14C). In this case the policy is phased in over five years ($g$ = 0.2).

What are the consequences of lags for policy choice? With lag, the utility of the optimal policy, the mean P concentration of the lake, and the mean input level are all reduced (Fig. 15). Because of the fluctuations, the optimal policy reduces P input rate, thereby reducing the mean P concentration (although year-to-year variability is high) and foregoing some of the potential utility. Therefore, if there are lags in implementing policy, then the optimal policy is more cautious with respect to P inputs. This general result holds for irreversible, hysteretic, or reversible dynamics.

Gradual implementation of the optimal policy

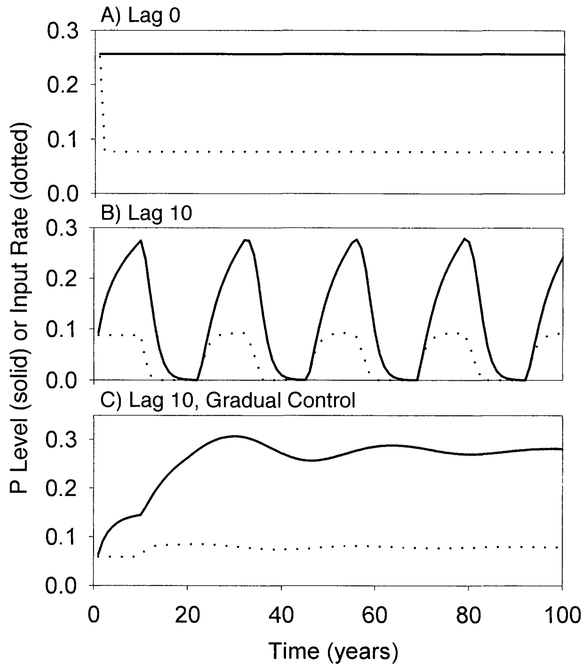


FIG. 14. P level (solid line, dimensionless) and input rate (dotted line, dimensionless) for three control scenarios. (A) No lag in implementing policy that maximizes discounted net present value. (B) 10-yr lag in implementing policy, but the lag is ignored in calculating the policy that maximizes discounted net present value. (C) 10-yr lag in implementing policy is ignored in calculating the policy that maximizes discounted net present value, but the policy is phased in gradually over five years. Parameters for simulations: $b = 0.52$, $q = 2$, $\alpha_0 = 2$, $\beta_1 = 0.4$, $\delta = 0.98$.

achieves results that are almost identical to those obtained when the lag is zero (Fig. 15). Therefore, gradual implementation compensates effectively for lags.

Results depicted in Fig. 15 were calculated for a lake that is initially at a low level of phosphorus ($X = 0.1$). The same general pattern holds for eutrophic lakes that are reversible or hysteretic. For eutrophic lakes that are irreversible, changes in P input policy alone are relatively ineffective.

This result has important implications for managers setting P input policies for lakes in relatively undeveloped watersheds. Such lakes are likely to be relatively oligotrophic with a diversity of potential dynamics under enrichment. Management will seek policies that allow development, agriculture, aquaculture, and so forth to generate P inputs up to the level that maximizes the overall net utility of these activities and lake ecosystem services. Lags in choosing and implementing these policies will be inevitable. Managers can compensate for lags by gradually phasing in the increased P inputs. If gradual policy implementation is not possible, then substantially lower targets for P inputs must be set to maximize the utility.

## Uncertain Response of Lake to Changes in P Inputs

Our ability to predict lake responses to P input policies is imperfect. Although we are certain that P input is a fundamental cause of lake eutrophication (Schindler 1977, Smith 1998), predictions of lake eutrophication models exhibit considerable variability (Vollenweider 1976, Reckhow 1979). Sometimes lakes fail to respond to P management as predicted by limnological models (Larsen et al. 1979, Ryding 1981, Cullen and Forsberg 1988, Jeppesen et al. 1991). Even for experimental lakes and lakes for which we have extensive long-term data, the scatter of data around empirical models cannot be ignored (Schindler et al. 1978, Schindler et al. 1987, Carpenter et al. 1998*b*, Lathrop et al. 1998). The variance of this scatter is related to the uncertainty we wish to consider here.

In the dimensionless form of our model, uncertainty about lake response to P input policies is represented by the statistical distribution of the parameter *b*. We have already seen that this parameter has strong effects on policy choice. We now consider how well *b* may be known, and the consequences of uncertainty about *b*.

### *Magnitude of ecological uncertainty*

To illustrate the magnitude of uncertainty about *b*, we use data from one of the world's best-studied lakes,

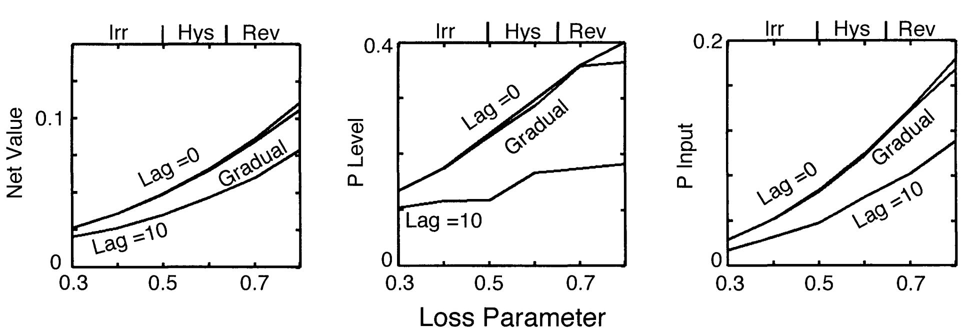


FIG. 15. Utility (net value), P level (dimensionless), and P input rate (dimensionless) vs. the loss parameter *b* for lags of 0 or 10 yr, and for a lag of 10 yr with gradual policy implementation over a 5-yr period. Each policy maximizes discounted net present value. Parameters: $q = 2$, $\alpha_0 = 2$, $\beta_1 = 0.4$, $\delta = 0.98$. Abbreviations: Irr, irreversible; Hys, hysteretic; Rev, reversible.

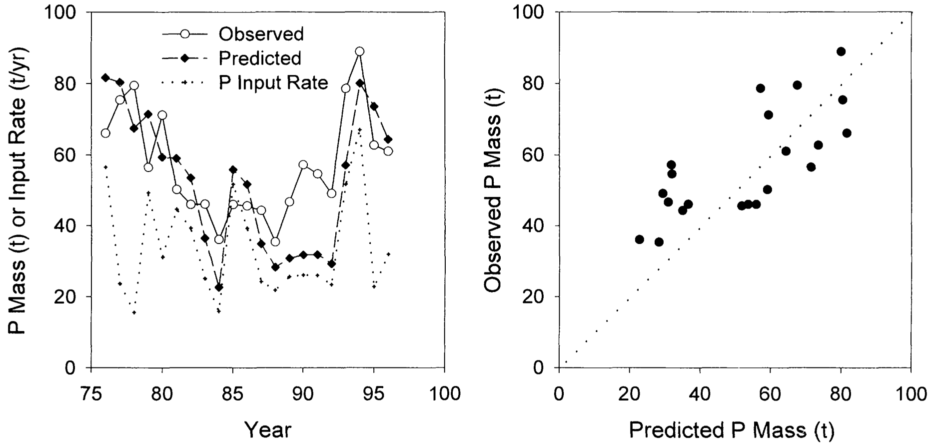


FIG. 16. Left: time series of annual P mass (in metric tons [t], where 1t = $10^6$ g) and input rate for Lake Mendota, Wisconsin, as measured by Lathrop et al. (1998) and predicted by the model. Right: observed vs. predicted P mass in the lake. Diagonal dotted line shows predictions = observations.

Lake Mendota, Wisconsin, USA. For this lake we have 21 yr of detailed, consistent measurements of P input rate and P mass (Lathrop et al. 1998). Independent studies have shown that internal recycling of P from the hypolimnion is substantial during periods of anoxia (Stauffer 1987, Soranno et al. 1997). We fit a discrete form of the model to the time series of $P$ and $l$ (Lathrop et al. 1998):

$$P_{t+1} = P_t + l_t - sP_t + \frac{rP_t^q}{m^q + P_t^q}. \quad (9)$$

The parameters $s$, $r$, $m$, and $q$ were estimated by the observation error procedure of Walters (1986) using least squares as the loss function and the simplex method to find the optimum estimates (Nelder and Mead 1964). We estimated the proportion of parameter sets that led to reversible, hysteretic, or irreversible dynamics by bootstrapping. To generate bootstrap estimates, pseudovalues were created by adding randomly selected residuals to predictions, then estimating $s$, $r$, and $m$ while fixing $q$ at the point estimate (Efron and Tibshirani 1993).

The model fit is presented in Fig. 16. There was a tendency for observations to exceed predictions when P input rates were relatively low. For example, during the period 1988–1992 inputs were relatively low and predictions were consistently lower than observations. This pattern suggests that recycling may be larger than estimated by the model. Diagnostic plots of residuals (normal probability plot, autocorrelation function, partial autocorrelation function) suggested no departures from model assumptions although with 21 data points the power to detect autocorrelations is low. Estimates of the parameters were $s = 0.817$, $r = 731\,000$ kg/yr, $m = 116\,000$ kg, and $q = 7.88$. These estimates seem reasonable in view of the P retention rates reported by Lathrop et al. (1998) and recycling rates reported by Soranno et al. (1997).

Despite more than two decades of high quality data and a good fit of the model, the future dynamics of Lake Mendota are uncertain. Bootstrapping showed considerable variability in the parameters, with a range of >100-fold for $m$ and >10-fold for $r$ (Fig. 17). In addition, the bootstrap distributions appear to be bimodal. This bimodality may indicate that two distinct dynamic patterns existed in Lake Mendota over the period of record, but the data are not sufficient to explore this possibility in detail.

Our analyses suggest that Lake Mendota may not be restorable by reducing P inputs alone. The mean estimates of the parameters indicate that the lake is irreversible, but these estimates have significant variability. The distribution of steady states was estimated from the bootstrapped distributions of $m$, $r$, and $s$. We found a 57.7% probability that the lake is reversible, but a 20.8% probability that the lake is irreversible and a 21.5% probability that the lake is hysteretic. Although the lake has responded to past variations in P input (Lathrop et al. 1998), recycling is substantial (Soranno et al. 1997). Thus there is ample reason for uncertainty about the lake's response to future changes in P input. Recognizing this uncertainty, recent management plans for Lake Mendota have included both reductions of P input and biomanipulation (Kitchell 1992, Lathrop et al. 1998).

Lake Mendota represents a "best-case scenario" for minimizing uncertainty. Yet, the distribution of outcomes that must be considered for Lake Mendota ranges from irreversible dynamics to reversible dynamics. Most management situations will be even more uncertain. The diversity of experiences with P reduction programs suggests that a full range of situations, from reversible to irreversible, is possible.

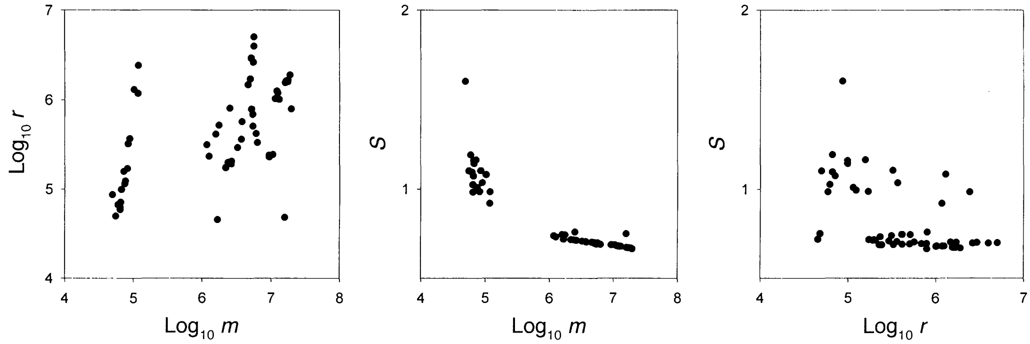


FIG. 17. Scatterplots of bootstrapped parameter values ($s$, $\log_{10}m$, $\log_{10}r$) from the fit of the model to observations of Lake Mendota, Wisconsin. Each panel shows 100 randomly chosen bootstrap estimates. For bootstrapping, $q$ was fixed at the optimal value 7.88.

### *Policy choice with ecological uncertainty*

What are the consequences of uncertainty about $b$ for the optimal policy? We compared a "naive" policy, which assumes $b$ is known precisely and the lake is hysteretic ($b = 0.54$) with an "informed" policy in which $b$ is uncertain. The assumed Student's $t$ distribution of $b$ has mean 0.54 and standard deviation 0.18. This distribution was chosen arbitrarily to span the range of interesting dynamics.

The naive optimal policy tends to overestimate utility and underestimate the P concentrations that management can achieve (Fig. 18). The naive policy tends to overestimate the P input rate when P levels are low, and underestimate the P input rate when P levels are high. At high P levels, the optimal policy under uncertainty averages over the possibility that the lake is reversible, in which case some input is tolerable, and the possibility that the lake is irreversible, in which case it is impossible to restore the lake but some utility can be gained from polluting activities.

These results indicate that policies based on the assumption that lake dynamics are precisely known will underestimate attainable P concentrations, and overestimate economic value, in comparison with policies that account for uncertainty. For lakes with low P levels, naive policies will set P input rates that are too high. For lakes that are already eutrophic, naive policies may actually be more conservative than informed policies with respect to P input rates, because the naive policy assumes that low P inputs will restore the lake. In contrast, the informed optimal policy maximizes expected utility over extremely different possibilities: that the lake is reversible or irreversible. An experimental policy could be used to assess whether the lake is reversible or irreversible. For example, if inputs are drastically reduced for a period of years and P levels decline rapidly, the lake is apparently reversible. The opposite experiment, increasing P inputs for a period of years, is more risky because the lake may prove irreversible.

## IMPACT OF DISCOUNTING

The discount factor $\delta$ determines the weight attached to future benefits. A low discount factor will favor policies that increase P input in the present and risk eutrophication of the lake in the future, whereas a high discount factor will give greater weight to sustaining the lake in the low-P state even if present benefits of polluting activities must be curtailed. Because of this powerful effect on policy choice, the selection of discount factors is a great source of controversy in environmental accounting and economics (Brock 1977, Chichilnisky 1997, Heal 1997). Also, some critics doubt that discounting is appropriate for ecosystems. The argument for discounting the value of money over time is based on human behavior (Freeman 1993, Hanley and Spash 1993, Dixon et al. 1994). It seems reasonable to express ecosystem services in monetary terms, to expose hidden values and add another dimension for environmental decision making (Daily 1997). It does not follow, however, that ecosystems or their services should be discounted over time in the same way that money is. Here we cannot resolve these fundamental concerns, but we can demonstrate the impact of discounting on lake management policies.

The discount factor interacts strongly with stochasticity in P inputs and uncertainty in lake dynamics. For example, compare a risky optimal policy, which assumes a lake is reversible, with a cautious optimal policy which assumes a lake is irreversible, under strong ($\delta = 0.9$) and weak ($\delta = 0.99$) discounting. Under the cautious policy, P input rates will be lower (Fig. 19A). Under both the cautious and risky policies, weak discounting leads to lower P input rates because the future state of the lake carries more weight. Now suppose the lake is actually irreversible. Implementation of the risky policy will yield lower utilities than the cautious policy, because the risky policy shifts the lake into a lower-utility, high-P state (Fig. 19B). The loss due to adopting the risky policy is the difference between the utility curves (Fig. 19C). The loss is large at low P

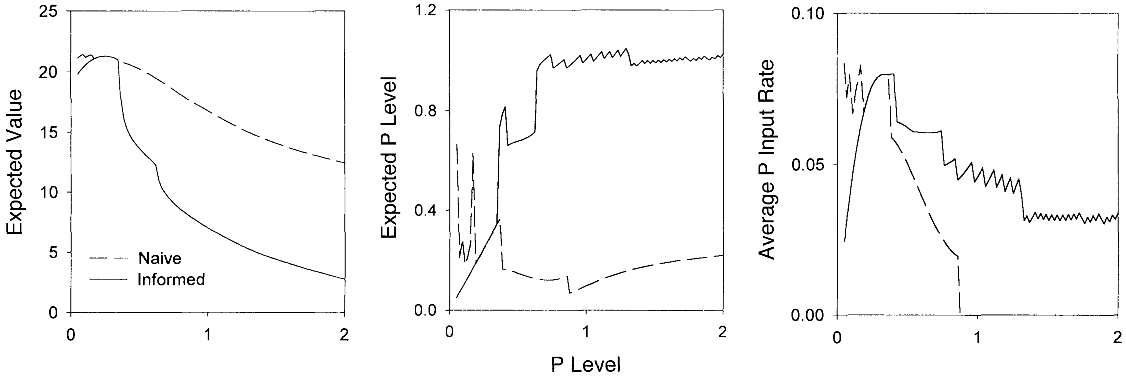


FIG. 18. Expected utility (value), P level (dimensionless), and P input rate (dimensionless) vs. initial P level (dimensionless) for naive policy (assumes that the sedimentation parameter $b$ is precisely known and the lake is hysteretic, $b = 0.54$) and informed policy (assumes $b$ is uncertain, mean = 0.54, $\sigma = 0.18$). Naive results maximize discounted net present value assuming $b = 0.54$. Informed results maximize net present value given the distribution of $b$. Other parameters: $q = 2$, $\alpha_0 = 2$, $\beta_1 = 0.4$, $\delta = 0.95$.

levels, where appropriate P inputs can prevent irreversible change, and small at high P levels where the lake is irreversibly eutrophic regardless of policy choice. The loss also depends on the discount factor used to calculate the policies. At low P levels, the loss is worse under weak discounting because the future state of the lake is weighted heavily. At intermediate P levels, the loss is worse under strong discounting because target P inputs are large in the critical range where lower inputs might prevent the shift to the irreversible high-P state. At high P levels, the lake is already irreversibly eutrophic regardless of policy choice, and losses are similar regardless of the discount factor. A similar interaction is discussed for linear models by Brock (1977).

The discount factor will also affect the decision maker's willingness to invest in research to determine whether the lake is reversible or irreversible. Such research will lead to increased future utilities. If future utilities are discounted heavily, however, the payoff from the research declines (Walters 1986). Weak discounting ($\delta$ near 1), in contrast, tends to favor research to improve predictions of lake response to future P inputs.

Alternative approaches to distributing benefits over time, which compromise between discounting the future and the need to sustain the ecosystem indefinitely, have been considered (Beltratti et al. 1996, Chichilnisky 1996, 1997, Heal 1996). These approaches would yield policies similar to the weak discount factor case analyzed here. They would lead to cautious P input

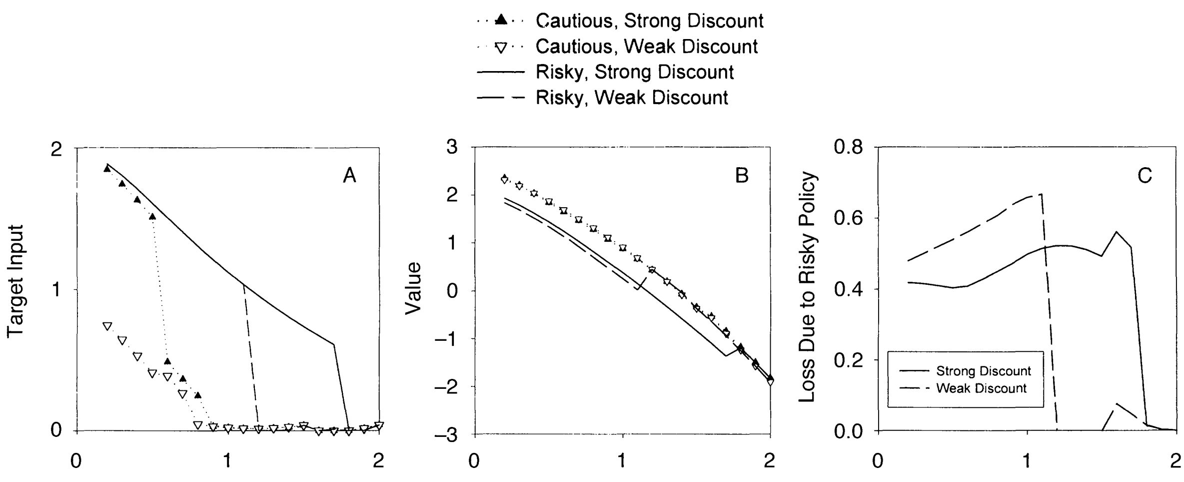


FIG. 19. (A) Target P input rate (dimensionless), (B) utility (value), and (C) loss due to adopting the risky policy vs. P level (dimensionless) for strong discounting ($\delta = 0.9$) and weak discounting ($\delta = 0.99$). The cautious policy assumes that the lake is irreversible ($b = 0.42$). The risky policy assumes that the lake is reversible ($b = 0.62$). Parameters: $b = 0.42$, $q = 2$, $\alpha_0 = 1$, $\beta_1 = 0.5$, $\sigma = 0.25$, df = 10.

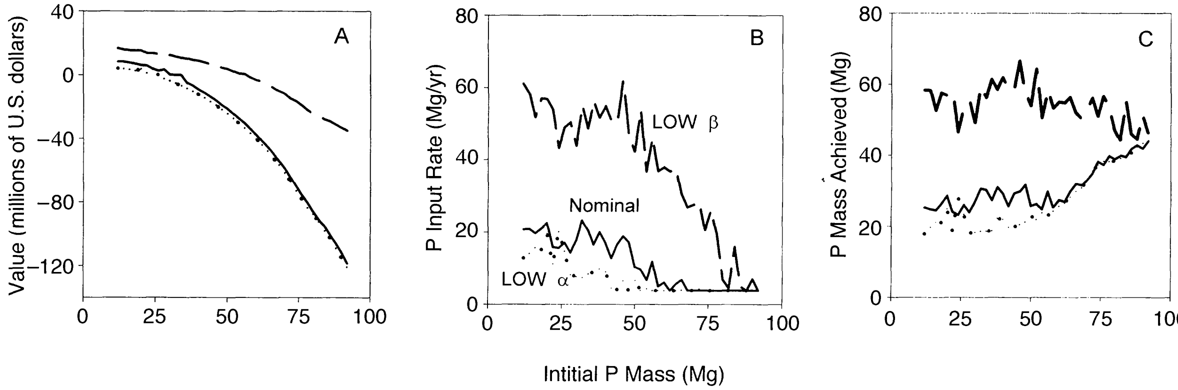


FIG. 20. Results of analysis of nominal (solid line), low demand or low α (dotted line), and low compensation or low β (dashed line) scenarios for management of Lake Mendota, Wisconsin. In all panels, the abscissa is initial P mass in the lake (Mg) before management, and curves show the policy that maximizes net discounted present value (given $\delta = 0.95$) for the specified scenario. (A) Net discounted present value (millions of U.S. dollars). (B) P input rate (Mg/yr). (C) Expected P mass in the lake (Mg).

rates, higher losses if the wrong policy is selected when P levels are low, and strong incentives for learning to predict the response of the lake more accurately.

## IMPACT OF OTHER ECONOMIC PARAMETERS

What are the effects of the utility functions for P emissions and the level of P in the lake? Here we present sample calculations that show the effects of alternative utility functions on policies for managing Lake Mendota. These calculations were done in natural, not dimensionless, units using parameters for the lake presented above and the utilities explained in Appendix B (Appendix E). Economic data are insufficient for a full uncertainty analysis of the economic parameters, but we can illustrate the effects of the parameters by exploring alternative scenarios.

We consider three scenarios: a nominal scenario based on the information described in Appendix B, a low demand (or low α scenario) based on the assumption that all remediable P inputs derive from agriculture so $\alpha_0$ is only U.S. \$132/kg P (Appendix B); and a low compensation (or low β) scenario in which the proposed payment for remediation is only U.S. \$10 $\times$ $10^6$ (leading to $\beta_0$ = U.S. \$89 per kilogram of P and $\beta_1$ = U.S. \$5.14 $\times$ $10^{-2}$ per square kilogram of P by the formulae of Appendix B).

In the nominal scenario, a positive net discounted present value declines with initial P mass (Fig. 20A). The more degraded the lake, the greater the cost of restoration. The P input rate that maximizes net present value declines from ~20 000 kg/yr when P levels are low to the minimum possible input rate (3800 kg/yr) when P levels are high (Fig. 20B). The expected P mass in the lake under these input rates increases with initial P mass (Fig. 20C), because of recycling of the P that is already present in the lake. According to the nominal scenario, the present state of the lake (57 000 kg) represents a substantial economic loss. The nominal scenario suggests that the lake could perhaps be restored to a condition of positive net present value. This might be achieved by drastic input reductions (to ~10 000 kg/yr or less) for long enough to reduce P mass in the lake below ~25 000 kg/yr.

In the low demand scenario, values are diminished because the calculations include only the P and profits from agriculture, omitting the role of development which generates more profit per unit pollution. P input rates that maximize net present value are even more drastic than in the nominal scenario (Fig. 20B). Because agricultural profits are relatively low per unit pollution, the low demand scenario suggests that economic factors favor more stringent reductions in pollution than the nominal scenario.

In the low compensation scenario, the economic losses caused by high P levels are assumed to be lower. Consequently, in comparison with the nominal scenario discounted net present values are higher (Fig 20A), target input rates are higher (Fig 20B), and values are maximized by maintaining higher P levels in the lake (Fig. 20C). In this case, the economically optimal policy makes the lake eutrophic because the profits per unit pollution are large relative to the lost amenities associated with the lake's condition.

This comparison shows that ecological outcomes are extremely sensitive to relatively modest changes in economic parameters. In many cases the economic parameters will not be narrowly specified by the available data. In such situations, analyses like Fig. 20 will not provide unambiguous answers to policy questions. Opposing interest groups could easily argue for economic parameter sets that favored a particular outcome. Moreover, the economic parameters are assumed to be static in this (and most) analyses. In fact, we can easily envision situations in which economic parameters evolve over time. For example, the Lake Mendota watershed is slowly changing from agricultural to urban land uses

(Soranno et al. 1996). This trend may increase the number of lake users (and thereby the βs) while decreasing agricultural profits (thereby reducing one component of α), perhaps supporting the argument for input reductions. Social dynamics of this sort greatly complicate the problem of environmental policy analysis.

## Discussion

An important lesson from this analysis is a precautionary principle. If P inputs are stochastic, lags occur in implementing P input policy, or decision makers are uncertain about lake response to altered P inputs, then P input targets should be reduced. In reality, all of these factors—stochasticity, lags, uncertainty—occur to some degree. Therefore, if maximum economic benefit is the goal of lake management, P input targets should be reduced below levels derived from traditional deterministic limnological models. The reduction in P input targets represents the cost a decision maker should be willing to pay as insurance against the risk that the lake will recover slowly or not at all from eutrophication. This general result resembles those derived in the case of harvest policies for living resources subject to catastrophic collapse (Ludwig 1995).

This paper demonstrates how the appropriate P inputs could be calculated for any lake. The specific targets will differ among lakes. In general, targets will be lower for lakes that are thought to be irreversible, lakes that are subject to highly variable inputs, situations where there will be a significant lag between perception of a problem and management action, and lakes with more uncertain dynamics. In many cases, uncertainty can be reduced by limnological research and careful modeling, but the case of Lake Mendota makes it clear that the remaining uncertainty can be substantial even when excellent long-term data are available. Uncertainty is exacerbated by the possibility that the model structure may omit processes that turn out to be important in the future. For example, changes in sediments over decades may alter recycling and thereby the reversibility of eutrophication. Thus the reversibility term $b$ (Eq. 6) becomes a dynamic variable, not a parameter. Such structural uncertainties, combined with substantial errors in parameter estimates, should lead to substantial caution in setting P input targets for lakes.

Limnologists have been effective in communicating a sound fundamental understanding of the eutrophication process. They have been far less successful in communicating the risks of slow recovery from eutrophication, or the possibility that eutrophication cannot be reversed by reducing nutrient inputs alone. Responses to P controls will be uncertain even for lakes that have been subjected to long-term studies of the highest quality. This does not mean that limnological studies have little to offer. On the contrary, limnological data offer the best evidence for the need to control P inputs and the feasibility of various other interventions that may control eutrophication (Cooke et al. 1993). However, accurate statements of uncertainty are a crucial element of a limnological analysis that is useful for policy making. What we do not, or cannot, know is crucial to policy and among the strongest arguments for caution in setting P input targets.

We believe that the precautionary principle which emerges from our model applies to a wide range of scenarios in which maximum economic benefit is sought from an ecosystem subject to hysteretic or irreversible changes. However, a great deal of additional work is required to apply our general approach to any specific decision-making situation. For example, costs and benefits of lake management strategies other than P management are not explicit in the model. The model collapses a great deal of ecological and economic information into a few parameters, thereby gaining clarity while sacrificing detail that may be important in particular applications. The general approach outlined here, combined with numerical simulations, should be adaptable to a wide range of situations but this will entail additional work and considerable site-specific data.

The greatest barrier to implementing our model for a specific project is likely to be economic data. The available data are few and highly specific to particular sites, times, and impacts (Wilson and Carpenter 1999). Data will often be insufficient to estimate economic uncertainties or project changes in economic parameters over time. In the absence of credible data for economic parameters, economic policy analyses for lake management will be largely a hypothetical exercise. Even hypothetical exercises may reveal benefits or costs that have been ignored or underestimated, contrast management scenarios in useful ways, or demonstrate the need for caution in setting input targets.

An important limitation of our model is its highly simplified representation of social dynamics. Abstract utility functions are helpful for gaining the general insights reported here, but fall well short of capturing the trends, shocks, and nonlinearities that arise in social systems and may impact notions of utility or alter decisions through processes that have nothing to do with economic policy choice (Holling and Sanderson 1996). Exploring the interactions of nonlinear evolving social and ecological systems is a research frontier that may be glimpsed, at great distance, from the present analysis.

Like all economic analyses, our model is strictly utilitarian (Goulder and Kennedy 1997). It does not consider any nonutilitarian criteria for decision making, except as they may be reflected in social preferences measured by the economic parameters. As a consequence, anything that we do not know about the utility of lakes is excluded from the analysis. This ignorance could be viewed as a form of uncertainty, and as such it would increase the degree of precaution prescribed by the model (Heal et al. 1996). Alternatively, one could seek an ethical alternative to utilitarian criteria. Aldo Leopold, in a famous pair of essays entitled *The Conservation Ethic* (1933) and *Conservation Econom-*

*ics* (1934), argued that ethical criteria for environmental decisions were adaptive in an evolutionary sense, and in critical need of scientific investigation. This remains true today.

Lakes are a microcosm of a much broader range of problems in ecosystem management. Most attempts at ecosystem management seek the policies that derive greatest utility for an adapting, diverse society from evolving ecosystems subject to time lags, hysteresis, and potentially irreversible change. This is a problem of enormous complexity and utmost importance. In lakes, the spatial arena is fixed (watershed and lake boundaries), the fundamental natural science is well developed, many of the ecological, economic and social issues are well-defined, the scales in space and time are tractable, and experimental management programs can be replicated among lakes on the landscape. Lake management is a laboratory for analysis and management of evolving social-ecological systems.


## Acknowledgments

Criticisms and insights from Marten Scheffer, Dick Lathrop, Jeff Houser, and two anonymous referees significantly improved the model and presentation. We thank R. C. Bishop for discussions of valuation studies. We are grateful for the support of the Resilience Network, a Pew Fellowship to SRC, the North Temperate Lakes LTER site, the University of Wisconsin Sea Grant Institute (National Sea Grant College Program, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, and the State of Wisconsin, through federal grant NA90AA-D-SG469, Project 41), NSERC of Canada under grant #A-9239 to DL, and NSF (SBR 94 22670) and Vilas Trust support to WAB.


## Literature Cited


Baines, S. B., and M. L. Pace. 1994. Sinking fluxes across lakes spanning a trophic gradient: patterns and implications for the fate of planktonic primary production. Canadian Journal of Fisheries and Aquatic Sciences **51**:25–36.

Beltratti, A., G. Chichilnisky, and G. Heal. 1998. Sustainable use of renewable resources. Pages 49–76 *in* G. Chichilnisky, G. M. Heal, and A. Vercelli, editors. Sustainability: dynamics and uncertainty. Kluwer, Dordrecht, The Netherlands.

Betz, C. R., M. L. Lowndes, and S. Porter. 1997. Nonpoint source control plan for the Lake Mendota Priority Watershed Project. Wisconsin Department of Natural Resources Publication Number WT-481-97, Madison, Wisconsin, USA.

Brock, W. A. 1977. A polluted golden age. Pages 441–461 *in* V. Smith, editor. Economics of natural and environmental resources. Gordon and Breach, New York, New York, USA.

Bromley, D. W. 1990. The ideology of efficiency: searching for a theory of policy analysis. Journal of Environmental Economics and Management **19**:86–107.

Canfield, D. E., and R. W. Bachmann. 1981. Prediction of total phosphorus concentrations, chlorophyll *a,* and Secchi depths in natural and artificial lakes. Canadian Journal of Fisheries and Aquatic Sciences **38**:414–423.

Caraco, N. F., J. J. Cole, and G. E. Likens. 1991. A cross-system study of phosphorus release from lake sediments. Pages 241–258 in J. J. Cole, G. Lovett, and S. Findlay, editors. Comparative analyses of ecosystems. Springer-Verlag, New York, New York, USA.

Carpenter, S. R., N. F. Caraco, D. L. Correll, R. W. Howarth, A. N. Sharpley, and V. H. Smith. 1998*a*. Nonpoint pollution of surface waters with phosphorus and nitrogen. Ecological Applications **8**:559–568.

Carpenter, S. R., J. J. Cole, J. F. Kitchell, and M. L. Pace. 1998*b*. Impact of dissolved organic carbon, phosphorus and grazing on phytoplankton biomass and production in experimental lakes. Limnology and Oceanography **43**:73–80.

Carpenter, S. R., and K. L. Cottingham. 1997. Resilience and restoration of lakes. Conservation Ecology **1**(1):2. [on line: ⟨http://www.consecol.org/vol1/iss1/art2⟩].

Carpenter, S. R., C. E. Kraft, R. Wright, X. He, P. A. Soranno, and J. R. Hodgson. 1992. Resilience and resistance of a lake phosphorus cycle before and after food web manipulation. American Naturalist **140**:781–798.

Carson, R. T., and R. C. Mitchell. 1993. The value of clean water: the public's willingness to pay for boatable, fishable and swimmable quality water. Water Resources Research **29**:2445–2454.

Charlton, M. N. 1980. Hypolimnion oxygen consumption in lakes: discussion of productivity and morphometry effects. Canadian Journal of Fisheries and Aquatic Sciences **37**: 1531–1539.

Chichilnisky, G. 1996. Sustainable development: an axiomatic approach. Social Choice and Welfare **13**:231–257.

———. 1997. What is sustainable development? Land Economics **73**:467–491.

Cooke, G. D., E. B. Welch, S. A. Peterson, and P. R. Newroth. 1993. Restoration and management of lakes and reservoirs. Lewis Publishers, Boca Raton, Florida, USA.

Cornett, R. J., and F. H. Rigler. 1980. The areal hypolimnetic oxygen deficit: an empirical test of the model. Limnology and Oceanography **25**:672–679.

Cullen, P., and C. Forsberg. 1988. Experiences with reducing point sources of phosphorus to lakes. Hydrobiologia **170**: 321–336.

Daily, G., editor. 1997. Nature's services. Island Press, Washington, D.C., USA.

Dane County Regional Planning Commission. 1995. Dane County Water Quality Plan. Madison, Wisconsin, USA.

del Georgio, P. A., and R. H. Peters. 1994. Patterns in planktonic P:R ratios in lakes: influence of lake trophy and dissolved organic carbon. Limnology and Oceanography **39**: 772–787.

Dixon, J. A., L. Fallon, R. A. Carpenter, and P. B. Sherman. 1994. Economic analysis of environmental impacts. Earthscan Publications, London, UK.

Edmondson, W. T. 1991. The uses of ecology: Lake Washington and beyond. University of Washington Press, Seattle, Washington, USA.

Efron, B., and R. J. Tibshirani. 1993. An introduction to the bootstrap. Chapman and Hall, New York, New York, USA.

Freeman, M. A. 1993. The measurement of environmental and resource values: theory and methods. Resources for the Future, Washington, D.C., USA.

Frisk, T., J. S. Niemi, and K. A. I. Kinnunen. 1980. Comparison of statistical phosphorus retention models. Ecological Modelling **12**:11–27.

Goulder, L. H., and D. Kennedy. 1997. Valuing ecosystem services: philosophical bases and empirical methods. Pages 23–48 in G. Daily, editor. Nature's services. Island Press, Washington, D.C., USA.

Gunderson, L. H., C. S. Holling, and S. S. Light. 1995. Barriers and bridges to the renewal of ecosystems and institutions. Columbia University Press, New York, New York, USA.

Hanley, N., and C. L. Spash. 1993. Cost-benefit analysis and the environment. Edward Elgar Publishing, Cornwall, UK.

Heal, G. M. 1997. Discounting and climate change. Climatic Change **37**:335–343.

———. 1998. Interpreting sustainability. Pages 3–22 *in* G. Chichilnisky, G. M. Heal, and A. Vercelli, editors. Sustainability: dynamics and uncertainty. Kluwer, Dordrecht, The Netherlands.

Heal, G., G. Chichilnisky, and A. Beltratti. 1998. Uncertain future preferences and conservation. Pages 257–275 *in* G. Chichilnisky, G. M. Heal, and A. Vercelli, editors. Sustainability: dynamics and uncertainty. Kluwer, Dordrecht, The Netherlands.

Holling, C. S. 1973. Resilience and stability of ecological systems. Annual Review of Ecology and Systematics **4**:1–23.

Holling, C. S., and G. K. Meffe. 1996. On command-and-control, and the pathology of natural resource management. Conservation Biology **10**:328–337.

Holling, C. S., and S. Sanderson. 1996. Dynamics of (dis)harmony in ecological and social systems. Pages 57–86 in S. S. Hanna, C. Folke, and K. G. Mäler, editors. Rights to nature. Island Press, Washington, D.C., USA.

Houser, J. N. 1998. Food web structure and experimental enrichment: effects on phosphorus sedimentation and resuspension. Thesis. University of Wisconsin, Madison, Wisconsin, USA.

Jeppesen, E. P. Kristensen, J. P. Jensen, M. Søndergaard, M. Mortensen, and T. Lauridsen. 1991. Recovery resilience following a reduction in internal phosphorus loading of shallow, eutrophic Danish lakes: duration, regulating factors, and methods for overcoming resilience. Memorie Istituto Italiano di Idrobiologie **48**:127–148.

Kitchell, J. F., editor. 1992. Food web management: a case study of Lake Mendota. Springer-Verlag, New York, New York, USA.

Kuznetsov, Y. A. 1995. Elements of applied bifurcation theory. Springer-Verlag, New York, New York, USA.

Larsen, D. P., D. W. Schults, and K. Malueg. 1981. Summer internal phosphorus supplies in Shagawa Lake, Minnesota. Limnology and Oceanography **26**:740–753.

Larsen, D. P., J. van Sickel, K. W. Malueg, and D. P. Smith. 1979. The effect of wastewater phosphorus removal on Shagawa Lake, Minnesota: phosphorus supplies, lake phosphorus and chlorophyll *a*. Water Research **13**:1259–1272.

Lathrop, R. C., S. R. Carpenter, C. A. Stow, P. A. Soranno, and J. C. Panuska. 1998. Phosphorus loading reductions needed to control blue-green algal blooms in Lake Mendota. Canadian Journal of Fisheries and Aquatic Sciences, *in press*.

Leopold, A. 1933. The conservation ethic. Journal of Forestry **31**:634–643.

———. 1934. Conservation economics. Journal of Forestry **31**:537–544.

Lindley, D. V. 1985. Making decisions. John Wiley and Sons, New York, New York, USA.

Ludwig, D. 1995. A theory of sustainable harvesting. SIAM Journal of Applied Mathematics **55**:564–575.

Ludwig, D., B. Walker, and C. S. Holling. 1997. Sustainability, stability and resilience. Conservation Ecology **1**(1): 7 [on line: ⟨http://www.consecol.org/vol1/iss1/art7⟩].

Mortimer, C. H. 1941. The exchange of dissolved substances between mud and water in lakes. Journal of Ecology **29**: 280–329.

———. 1942. The exchange of dissolved substances between mud and water in lakes. Journal of Ecology **30**:147–201.

Nelder, J. A., and R. Mead. 1964. A simplex method for function minimization. Computer Journal **7**:308–313.

Novotny, V., and H. Olem. 1994. Water quality: prevention, identification and management of diffuse pollution. Van Nostrand Reinhold, New York, New York, USA.

NRC (National Research Council). 1992. Restoration of aquatic ecosystems: science, technology and public policy. National Academy Press, Washington, D.C., USA.

NRC (National Research Council). 1993. Managing wastewater in coastal urban areas. National Academy Press, Washington, D.C., USA.

Nürnberg, G. K. 1984. Prediction of internal phospohrus load in lakes with anoxic hypolimnia. Limnology and Oceanography **29**:135–145.

———. 1995. Quantifying anoxia in lakes. Limnology and Oceanography **40**:1100–1111.

Perrings, C., K. G. Mäler, C. Folke, C. S. Holling, and B. O. Jansson. 1995. Biodiversity loss: economic and ecological issues. Cambridge University Press, London, UK.

Postel, S. L., and S. R. Carpenter. 1997. Freshwater ecosystem services. Pages 195–214 *in* G. Daily, editor. Nature's Services. Island Press, Washington, D.C., USA.

Prepas, E. E., and J. M. Burke. 1997. Effect of hypolimnetic oxygenation on water quality in Amisk Lake, Alberta, a deep eutrophic lake with high internal phosphorus load rates. Canadian Journal of Fisheries and Aquatic Sciences **54**:2111–2120.

Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. 1989. Numerical recipes in C. Cambridge University Press, London, UK.

Reckhow, K. H. 1979. Empirical lake models for phosphorus: development, applications, limitations, and uncertainty. Pages 193–221 *in* D. Scavia and A. Robertson, editors. Perspectives on lake ecosystem modeling. Ann Arbor Science, Ann Arbor, Michigan, USA.

Reckhow, K. H., and S. C. Chapra. 1983. Engineering approaches for lake mangement. Volume I. Data analysis and empirical modeling. Butterworth, Boston, Massachusetts, USA.

Ripley, B. D. 1987. Stochastic simulation. John Wiley and Sons, New York, New York, USA.

Ryding, S. O. 1981. Reversibility of man-induced eutrophication: experiences of a lake recovery program in Sweden. International Revue der Gesampten Hydrobiologie **66**:189–207.

Sas, H. 1989. Lake restoration by reduction of nutrient loading: expectations, experiences, extrapolations. Academia Verlag, Richarz, St. Augustin, Germany.

Scheffer, M. 1998. Ecology of shallow lakes. Chapman and Hall, London, UK.

Scheffer, M., H. Hosper, M.-L. Meijer, B. Moss, and E. Jeppesen. 1993. Alternative equilibria in shallow lakes. Trends in Ecology and Evolution **8**:260–262.

Schindler, D. E., S. R. Carpenter, K. L. Cottingham, X. He, J. R. Hodgson, J. F. Kitchell, and P. A. Soranno. 1995. Food web structure and littoral zone coupling to pelagic trophic cascades. Pages 96–105 *in* G. A. Polis and K. O. Winemiller, editors. Food webs: integration of pattern and dynamics. Chapman and Hall, New York, New York, USA.

Schindler, D. W. 1977. Evolution of phosphorus limitation in lakes. Science **195**:260–262.

Schindler, D. W., E. J. Fee, and T. Rusczynski. 1978. Phosphorus input and its consequences for phytoplankton standing crop and production in the Experimental Lakes Area and in similar lakes. Journal of the Fisheries Research Board of Canada **35**:190–196.

Schindler, D. W., R. H. Hesslein, and M. A. Turner. 1987. Exchange of nutrients between sediments and water after 15 years of experimental eutrophication. Canadian Journal of Fisheries and Aquatic Sciences **44**:26–33.

Smith, V. H. 1998. Cultural eutrophication of inland, estuarine and coastal waters. Pages 7–49 *in* M. L. Pace and P. M. Groffman, editors. Successes, limitations and frontiers of ecosystem science. Springer-Verlag, New York, New York, USA.

Soranno, P. A., S. R. Carpenter, and R. C. Lathrop. 1997. Internal phosphorus loading in Lake Mendota: response to external loads and weather. Canadian Journal of Fisheries and Aquatic Sciences **54**:1883–1983.

Soranno, P. A., S. L. Hubler, S. R. Carpenter, and R. C. Lathrop. 1996. Phosporus loads to surface waters: a simple model to account for spatial patterns of land use. Ecological Applications **6**:865–878.
Stauffer, R. E. 1987. Vertical nutrient transport and its effect on phosphorus in 4 calcareous lakes. Hydrobiologia **154**: 87–102.
USEPA (U.S. Environmental Protection Agency). 1996. Environmental indicators of water quality in the United States. USEPA, Office of Water (4503F), U.S. Government Printing Office, Washington, D.C., USA. EPA **8410R-96-002**.
Vollenweider, R. A. 1976. Advances in defining critical loading levels for phosphorus in lake eutrophication. Memorie Istituto Italiano di Idrobiologia **33**:53–83.
Walters, C. J. 1986. Adaptive management of renewable resources. Macmillan, New York, New York, USA.
Wetzel, R. G. 1983. Limnology. W. B. Saunders, Philadelphia, Pennsylvania, USA.
Wilson, M. A., and S. R. Carpenter. 1999. Economic valuation of freshwater ecosystem services in the United States: 1971–1997. Ecological Applications **9**:772–783.
Wisconsin Department of Agriculture, Trade and Consumer Protection. 1995. Wisconsin 1994 Agricultural Statistics. Madison, Wisconsin, USA.

## APPENDIX A

### Bifurcation Analysis of the Lake Model

In continuous time the model is

$$\frac{dP}{dt} = l - sP + \frac{rP^q}{m^q + P^q}. \quad (A.1)$$

Integration of Eq. A.1 with a time step of one year suggests the following discrete-time approximation:

$$P_{t+1} = P_t e^{-s} + [l(1 - e^{-s})/s] + \frac{rP_t^q}{m^q + P_t^q}. \quad (A.2)$$

Note that this discrete time model with an annual time step cannot capture the seasonal timing of inputs and losses of phosphorus. Effects of seasonality could be included by using the continuous model directly, or by using the continuous model to derive an approximate discrete model that included seasonal inputs and losses. However, limnological analyses used in lake management are often based on annual budgets. Annual budget studies can be modeled using Eq. A.2 and we will adopt an annual time step for this paper.

We rescale the discrete-time model using

$$X = \frac{P}{m} \quad (A.3)$$

$$A = \frac{l}{m}\frac{1 - e^{-s}}{s} \quad (A.4)$$

$$B = 1 - e^{-s} \quad (A.5)$$

$$C = \frac{r}{m} \quad (A.6)$$

$$f(X) = \frac{X^q}{1 + X^q}. \quad (A.7)$$

With these definitions the discrete time model is written as

$$X_{t+1} = X_t + A - BX_t + Cf(X_t) \quad (A.8)$$

The equilibria of Eq. A.8 satisfy

$$F(X) = A - BX + Cf(X) = 0. \quad (A.9)$$

If we apply a small perturbation $X \rightarrow X + \delta X$ and $\delta$ is small, then to a first approximation

$$\delta X_{t+1} = \delta X_t\left(1 + \frac{\partial F}{\partial X}\right). \quad (A.10)$$

The equilibria are stable if

$$\left|1 + \frac{\partial F}{\partial X}\right| < 1. \quad (A.11)$$

In view of the definition of $F$, Eq. A.11 implies that

$$-2 < -B + Cf'(X) < 0. \quad (A.12)$$

The equality at the lower end ($B = 2$) is the condition for a

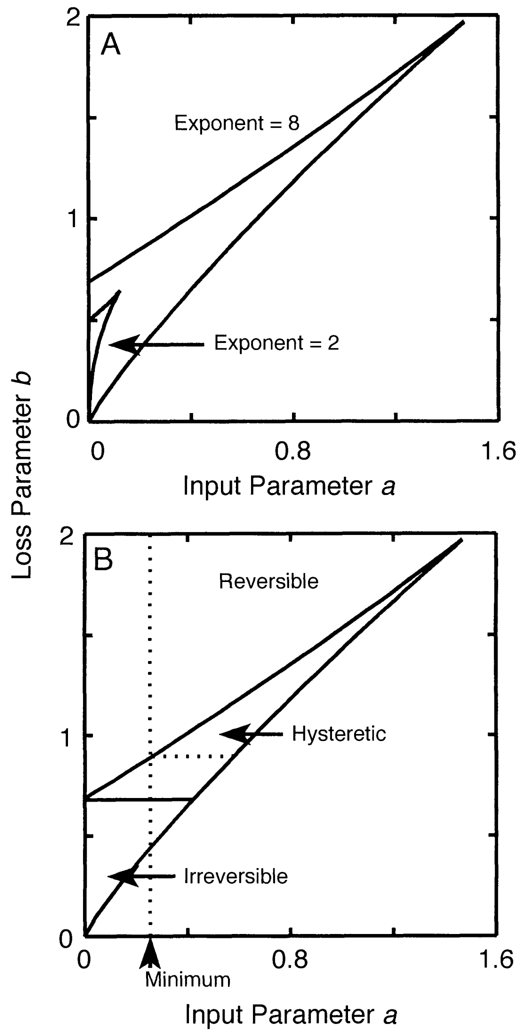


FIG. A1. (A) Bifurcation diagram for the dimensionless discrete-time model using values of 2 and 8 for the exponent $q$. The lines show the locus of double roots in parameter space ($b$ vs. $a$). Outside this locus there is one stable steady state; inside the locus there are two stable and one unstable steady states. (B) Parameter combinations that yield reversible, hysteretic, or irreversible dynamics for $q = 8$. The dotted lines show how these zones change if there is a positive minimum input rate.

period-doubling bifurcation (Kuznetzov 1995). This cannot occur in our case because $B < 1$ from Eq. A.5 and $f'(X) > 0$. The equality at the upper end, $B = Cf'(X)$, is the condition for double roots as the following argument shows.

To find the locus for double roots, we write the equation for the equilibria (A.9) as

$$bX - a = f(X) \tag{A.13}$$

where $a = A/C$ and $b = B/C$. This equation has a double root if the straight line $y = bx - a$ is tangent to the curve $y = f(X)$. The tangency occurs if Eq. A.13 is satisfied and also

$$b = f'(X). \tag{A.14}$$

The locus of parameter combinations for which the system has a double root is obtained by substituting Eq. A.14 into Eq. A.13:

$$a = Xf'(X) - f(X). \tag{A.15}$$

When Eq. A.15 is satisfied, the system has a double root (the point of tangency). Eqs. A.14 and A.15 are a parametric representation of a locus in parameter space (Fig. A1), which is the bifurcation diagram. Inside this locus, the system has three real roots. Two correspond to stable steady states, and one corresponds to an unstable steady state. Dynamics can be hysteretic or irreversible. Outside this locus, the system has only one real root, which corresponds to a stable steady state.

The area of parameter space corresponding to alternate states depends on the size of the exponent, $q$. Larger values of $q$ correspond to larger envelopes for alternate states (Fig. A1A).

Irreversible dynamics occur for values of $b$ (the loss parameter) less than the $y$-axis intercept of the upper margin of the envelope (Fig. A1B). For these values of $b$, $a$ (the input parameter) must become negative to shift the lake from the high-P steady state to the low-P steady state. Because $a$ cannot be negative, we say the lake is irreversible. If there is a positive minimum P input rate, the parameter combinations for irreversible and hysteretic dynamics shift to the right as shown by the dotted lines in Fig. A1B.

## APPENDIX B

### Estimation of Economic Parameters

Here we present a simple estimate of economic parameters to illustrate how the utility functions might be applied. See the references for more detailed descriptions (Carson and Mitchell 1993, Freeman 1993, Hanley and Spash 1993, Dixon et al. 1994, Goulder and Kennedy 1997, Wilson and Carpenter 1999). Data are for Lake Mendota, Wisconsin (Wisconsin Department of Agriculture, Trade, and Consumer Protection 1995, Dane County Regional Planning Commission 1995, Soranno et al. 1996, Betz et al. 1997, Lathrop et al. 1998; R. C. Bishop, *unpublished data*). Calculations presented here are for illustrative purposes only and are not meant to replace more detailed cost-benefit studies that are being undertaken for this lake.

To estimate the demand curve (Fig. 10A, but with abscissa P input in the natural units, kg/yr), we note that the P inputs of 34 000 kg/yr include an uncontrollable or baseline component of 3800 kg/yr (Lathrop et al. 1998). The remainder is potentially manageable, and comes largely from agriculture, with annual profits of U.S. $\$4 \times 10^6$ per year. If agriculture were the only source, we would calculate $\alpha_0 = \$4 \times 10^6/(34\,000 - 3800) \approx \$132$/kg P. We must also account for a lesser amount of P emitted by urban sources, primarily construction sites, which are more profitable. When these sites are included, we estimate $\alpha_0 \approx \$264$/kg P. We see no evidence for an inverse relationship between P emissions and the right to emit P to the lake, so we set $\alpha_1 = 0$.

To estimate the supply curve (Fig. 10B, but with abscissa P mass in the lake, in kilograms) we assume that the compensation required (or willingness to pay to restore) the lake would be zero if the P mass was at the ancestral level $P_a$. This gives us

$$\beta_0 = 2\beta_1 P_a. \tag{B.1}$$

Valuation studies of water quality typically measure the utility $C$, in dollars, of altering a lake from the present water quality (measured here as P mass $P_n$) to some specified goal for water quality ($P_g$). If we assume the lake is to be restored, so $P_g < P_n$, then

$$C = \int_{P_g}^{P_n} (2\beta_1 P - \beta_0)\, dP. \tag{B.2}$$

If we insert Eq. B.1 into B.2 and solve for $\beta_1$ we get

$$\beta_1 = \frac{C}{2P_a(P_g - P_n) - P_g^2 + P_n^2}. \tag{B.3}$$

Current management plans for Lake Mendota call for reductions of P mass in the lake from ~57 000 kg ($P_n$) to 28 500 kg ($P_g$). Valuation studies estimate the public's willingness to pay for these remediations ($C$) at about U.S. $\$30 \times 10^6$. We estimated $P_a$ (8671 kg) by solving Eq. 9 for the equilibrium P mass assuming that the ancestral input rate was 3800 kg/yr (Lathrop et al. 1998). This P mass lies within the range estimated by Soranno et al. (1996) for presettlement conditions. Using Eq. B.3, we estimate $\beta_1 \approx \$1.54 \times 10^{-2}/\text{kg}^2$ P. Inserting this value in Eq. B.1 we find $\beta_0 \approx \$268$/kg P.

## APPENDIX C

### Calculation of the Optimal Policy

The optimization problem is to find the sequence of P input rates $\mathbf{a}_t$ that maximizes

$$V(u) = \sum_{t=0}^{t=\infty} \delta^t [U_P(\mathbf{X}_t) + U_L(\mathbf{a}_t)] \tag{C.1}$$

subject to ecosystem dynamics as described in Appendix A. $\mathbf{X}$ is a vector of discretized phosphorus levels, and $\mathbf{a}$ is a vector of the P input rates chosen by the policymaker at each P level. It follows that the optimal policy maximizes

$$V(\mathbf{X}_t) = [U_P(\mathbf{X}_t) + U_L(\mathbf{a}_t)] + \delta\, E[V(\mathbf{X}_{t+1})]. \tag{C.2}$$

The expected value of $\mathbf{X}_{t+1}$ is determined by the policy choice $\mathbf{a}_t$. The optimal solution is computed iteratively as follows. (1) Choose an initial policy $\mathbf{a}^{(0)}$, for example set $\mathbf{a}$ to a vector of ones. (2) Calculate $V^{(0)}$ from Eq. C.2 using $\mathbf{a}^{(0)}$. (3) Given $V^{(0)}$, find the the vector $\mathbf{a}^{(1)}$ that maximizes the right-hand side of Eq. C.2. Call the corresponding solutions of C.2 $V^{(1)}$. (4) Given $V^{(1)}$, find the vector $\mathbf{a}^{(2)}$ that maximizes the right-hand side of C.2, and the corresponding solutions $V^{(2)}$. (5) Repeat these iterations until changes in $\mathbf{a}$ are smaller than some tolerance.

The optimal policies found by the iterative scheme are closely approximated by a "bang-bang" strategy which establishes a target P level $y$ and moves to that level as quickly as possible. Optimal policies from the iterative and bang-

bang methods are indistinguishable at the precision of our calculations. The bang-bang strategy is computed as follows. Choose $u(\mathbf{X}, y)$ such that

$$u(\mathbf{X}, y) = y - (1 - b)\mathbf{X} - f(\mathbf{X}). \quad (C.3)$$

This is the P input required to bring $\mathbf{X}$ to $y$ in a single step. But P inputs cannot be negative. Therefore we set

$$u_+(\mathbf{X}, y) = u(\mathbf{X}, y) \quad \text{if } u(\mathbf{X}, y) \geq 0$$

$$u_+(\mathbf{X}, y) = 0 \quad \text{if } u(\mathbf{X}, y) < 0. \quad (C.4)$$

We assume that it takes $\tau$ years to decide upon and implement a change in loading, but this lag is ignored in the decision-making process. Therefore:

$$a_t(y) = u_+(\mathbf{X}_{t-\tau}, y). \quad (C.5)$$

With these definitions, one can compute $V$ as a function of $y$ and find the value $y^*$ that maximizes $V$.

We also considered situations in which the optimal policy is implemented gradually. The target P level is reached over a number of years, rather than in a single time step. We replace Eq. C.3 with

$$u(\mathbf{X}, y, g) = \mathbf{X} + g(y - \mathbf{X}) - (1 - b)\mathbf{X} - f(\mathbf{X}) \quad (C.6)$$

where $g$ is a parameter that determines the fraction of the goal reached in a single step. If $g = 1$ then Eq. C.6 is identical to Eq. C.3.

## APPENDIX D

### Optimization of the Stochastic Model

In computing policies, we must account for the fact that the stochastic model,

$$X_{t+1} = X_t + u(X_t)e^z - bX_t + f(X_t) \quad (D.1)$$

yields a distribution of future values of $X$, rather than a point prediction. We handle this by introducing a mesh for X and calculating the probability of obtaining each possible value of $X_{t+1}$ given a particular value of $X_t$. The mesh is defined by discrete values $x_j$, $j = 0, \ldots, J$. Each mesh point $x_j$ is placed in an interval $[x_{j-}, x_{j+}]$ where

$$x_{j-} = (x_{j-1} + x_j)/2 \quad \text{and} \quad x_{j+} = (x_{j+1} + x_j)/2. \quad (D.2)$$

We define the probability of a transition from $x_j$ to $x_k$ as the probability corresponding to a transition from $X_t = x_j$ to a point in the interval $x_{k-} < X_{t+1} < x_{k+}$. This interval corresponds to the interval $W_{k-} < W < W_{k+}$ where

$$W_{k-} = \frac{1}{\sigma}\log\left[\frac{x_{k-} - x_j + bx_j - f(x_j)}{u(x_j)}\right] \quad (D.3)$$

and an analogous equation defines $W_{k+}$. $W$ is a deviate from a log $t$ distribution. The scale factor of the $t$ distribution is $\sigma$. If the expression $x_k - x_j + bx_j - f(x_j)$ is negative, then $x_k$ cannot be reached from $x_j$ in one step with nonnegative P input. In this case, the probability of $W_k$ is set to zero.

The probability of a transition from $x_j$ to $x_k$ is denoted by $C_{j,k}$. This probability is calculated as

$$C_{j,k} = \Pr[W_{k-} < W < W_{k+}]$$
$$= \Psi(W_{k+}, n) - \Psi(W_{k-}, n). \quad (D.4)$$

The Student's $t$ probability of $W$ with $n$ degrees of freedom, $\psi(W, n)$, is calculated from the incomplete beta function (Press et al. 1989).

Over one time step, the expected utility of a policy $u$ given an initial $x_j$ is

$$V(u, x_j) = U_{\mathrm{P}}(x_j) + U_{\mathrm{L}}(a_t)$$
$$+ \delta \sum_{k=0}^{J} C_{j,k}[U_{\mathrm{P}}(x_k) + U_{\mathrm{L}}(a_{t+1})]. \quad (D.5)$$

Utility functions for P level in the lake ($U_{\mathrm{P}}$) and loading ($U_{\mathrm{L}}$) are defined under *Economic framework for managing lake eutrophication: Model for economic utilities.* Note that $a_t$ depends on $x_j$ and $a_{t+1}$ depends on $x_k$ (Appendix C). Given a target P level, we calculate the input rate necessary to reach the target in one step (Appendix C). Eq. D.5 is solved for each candidate policy, and the policy with maximum utility is identified.

## APPENDIX E

### Calculation of Policies for Lake Mendota, Wisconsin

The optimal policy maximizes

$$V(P_t) = [U_{\mathrm{P}}(P_t) + U_{\mathrm{L}}(L_t)] + \delta E[U_{\mathrm{P}}(P_{t+1}) + U_{\mathrm{L}}(L_{t+1})] \quad (E.1)$$

(Appendix C) subject to P dynamics given by

$$P_{t+1} = P_t + (L_f + L_c)e^z - sP_t + \frac{rP_t^q}{m^q + P_t^q}. \quad (E.2)$$

Eq. E.2 is identical to Eq. 9 except that P inputs are divided into fixed ($L_f$) and controllable ($L_c$) components, and input disturbances follow a log Student's $t$ distribution as in Appendix D. For Lake Mendota, this $t$ distribution has scale factor 0.47 and df = 20 (Lathrop et al. 1998). Parameter distributions of Eq. E.2 were bootstrapped as in Fig. 17. For these calculations, we fixed $q$ at the point estimate (7.88) as in Fig. 17. Utilities are calculated as

$$U_{\mathrm{L}}(L_{c,t}) = \alpha_0 L_{c,t} \quad (E.3)$$

$$U_{\mathrm{P}}(P_t) = \beta_0 P_t - \beta_1 P_t^2 \quad (E.4)$$

with parameters as in Appendix B.

Following Appendix C, we follow a "bang-bang" strategy that seeks an target P level that maximizes $V(P)$. This converts the problem to searching in one dimension for the optimum. The input policy that reaches this optimal level in a single step is calculated, with the proviso that inputs cannot decrease below the minimal level $L_f$ = 3800 kg/yr (Lathrop et al. 1998). Given this policy, we can calculate the expectation in equation E.1 by sampling from the distribution of load disturbances (Ripley 1987) and the bootstrapped distribution of the parameters of Eq. 9. Eq. E.1 was solved for each candidate policy and the policy that maximized $V(P)$ was identified.